### Ohio Standards Connection:

**Geometry and Spatial Sense**

**Benchmark E**
Draw and construct representations of two- and three-dimensional geometric objects using a variety of tools, such as straightedge, compass, and technology.

**Indicator 6**
Draw nets for a variety of prisms, pyramids, cylinders and cones.

**Measurement**

**Benchmark B**
Use formulas to find surface area and volume for specified three-dimensional objects accurate to a specified level of precision.

**Indicator 4**
Derive formulas for surface area and volume and justify them using geometric models and common materials. For example, find:

- the surface area of a cylinder as a function of its height and radius;
- that the volume of a pyramid (or cone) is one-third of the volume of a prism (or cylinder) with the same base area and height.

### Lesson Summary:

In this four-part lesson, students explore three-dimensional objects (prisms, pyramids, cylinders and cones) to draw nets. Students use the understanding of drawing nets to find the surface area of pyramids and cylinders. Students discuss surface area by using visual representations of nets to derive formulas for the surface area of each object. Journal writing and “quick-write” prompts allow students to reflect on their understanding. Multiple entry points into the lesson serve students of different abilities. Part One acts as remediation or an introduction for students who have no prior experience with nets. Students who understand nets may begin with Part Two, Making Conjectures about Nets; or Parts Three and Four, Surface Area.

**Estimated Duration:** Two to three hours

### Commentary:

Mathematics Academic Content Standards for grades five to seven initially develop the concept of surface area of three-dimensional objects. In grade eight, students derive formulas for finding surface area. As students draw upon multiple senses in a learning episode, they more easily remember, recall and connect learning experiences. Using Nets to Find Surface Area captures students’ senses through building models and visual representations, discussing and supporting conjectures in collaboration with other students, reflecting upon learning and communicating in writing.

### Pre-Assessment:
- Distribute page one of the Hexomino Patterns, Attachment A.
- Ask students which of the hexominoes, when folded on the lines, makes a three-dimensional object. Allow students to analyze the four hexominoes.
- As students work, question individuals about how they came to their conclusions. Observe their strategies and evaluate understanding based on the following questions:
  - Does the student recognize nets and non-nets without folding?
  - Does the student need to fold the pattern to determine if it is a net?
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c. Does the student understand folding on the lines to create a three-dimensional object?

- For each hexomino, ask all students in the class to give you simultaneous visual cues (thumbs-up/thumbs-down) as to whether it can be folded into a three-dimensional object. Ask students to name or define two-dimensional patterns that can be folded into three-dimensional shapes (nets).

**Instructional Tip:**
If students struggle to meet the expectations of this task or demonstrate little understanding of nets, go to Instructional Procedures Part One, which provides students experience with nets. If students succeed or appear to have some background knowledge of nets, continue with the rest of the pre-assessment exercises and begin with Part Two of the lesson.

- Provide small groups of students a variety of three-dimensional models including triangular and rectangular prisms, triangular and rectangular pyramids, cones and cylinders. Direct students to sketch a net for the objects. Observe students and determine which students:
  a. Draw nets without assistance;
  b. Experience difficulty drawing one to two specified nets, or
  c. Experience difficulty drawing any net.

Use a checklist or take anecdotal notes describing student strategies/understanding.

- Collect the drawings of student work to determine how to proceed with instruction.

**Scoring Guidelines:**
Observe students to determine their understanding of identifying and defining “nets.” Record data and categorize students by level of understanding.
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**Post-Assessment:**

*Option 1*

Provide three-dimensional objects to small groups. When possible, use real-world objects such as cans, rectangular boxes, glass prisms and orange cones. Students draw nets of prisms (triangular and rectangular), pyramids (triangular and rectangular), cylinder and cone, and determine surface area of pyramids and cylinders.

**Scoring Guidelines:**

Use the rubric to score student work.

<table>
<thead>
<tr>
<th>Points</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 points</td>
<td>Draws nets for the four three-dimensional objects (prisms, pyramids, cones and cylinders).</td>
</tr>
<tr>
<td>3 points</td>
<td>Draws nets for three of the four three-dimensional objects (prisms, pyramids, cones and cylinders).</td>
</tr>
<tr>
<td>2 points</td>
<td>Draws nets for two of the four three-dimensional objects (prisms, pyramids, cones and cylinders).</td>
</tr>
<tr>
<td>1 point</td>
<td>Draws nets for one of the four three-dimensional objects (prisms, pyramids, cones and cylinders).</td>
</tr>
<tr>
<td>0 points</td>
<td>Draws all nets incorrectly or makes no attempt. Drawings do not show correct faces of the three-dimensional object.</td>
</tr>
</tbody>
</table>

*Option Two*

Use nets created by the students in previous lessons to create an assessment for recognizing nets and non-nets. Copy the drawings on paper for each student or copy onto a transparency and show the drawings on an overhead projector. Present 10 examples of nets and non-nets. Have students write responses on separate answer/notebook sheets, identifying three-dimensional object, and indicating if the drawing is or is not a net.

**Instructional Tip:**

For students who cannot identify nets, reduce the complexity and number of the three-dimensional figures used to assess drawing nets.

**Scoring Guidelines:**

Use the rubric to score student work.

<table>
<thead>
<tr>
<th>Points</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 points</td>
<td>Identifies 10 pattern shapes correctly, naming the three-dimensional object formed by the net and identifying non-nets.</td>
</tr>
<tr>
<td>3 points</td>
<td>Identifies eight to nine of the pattern shapes correctly, naming the three-dimensional object and identifying non-nets.</td>
</tr>
<tr>
<td>2 points</td>
<td>Identifies six to seven pattern shapes correctly, naming the three-dimensional object and identifying non-nets.</td>
</tr>
<tr>
<td>1 points</td>
<td>Identifies one to five pattern shapes correctly, naming the three-dimensional object and identifying non-nets.</td>
</tr>
</tbody>
</table>
Option Three

Provide students with models or common objects in the shape of cylinders and pyramids. Instruct them to find the surface area of each object. Explain the method for solving and provide reasoning for formulas used.

Scoring Guidelines:
Use the rubric to score student work.

<table>
<thead>
<tr>
<th>Score</th>
<th>Characteristics</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 points</td>
<td>Accurately finds surface area for cylinders and pyramids, and clearly explains reasoning for strategies and formulas used.</td>
</tr>
<tr>
<td>1 point</td>
<td>Accurately finds surface area and clearly explains reasoning for one of the two objects (pyramid or cylinder).</td>
</tr>
<tr>
<td></td>
<td>OR</td>
</tr>
<tr>
<td></td>
<td>Accurately finds surface area for both objects, but explanation is unclear, irrelevant or not provided.</td>
</tr>
<tr>
<td></td>
<td>OR</td>
</tr>
<tr>
<td></td>
<td>Finds surface area incorrectly, but explains reasoning for appropriate methods to find surface area.</td>
</tr>
<tr>
<td>0 points</td>
<td>Incorrectly calculates surface area.</td>
</tr>
<tr>
<td></td>
<td>Explanation provides little evidence of reasoning associated with the problem.</td>
</tr>
</tbody>
</table>

Instructional Procedures:
Part One  What are Nets?
1. Complete the pre-assessment activity. If students provide evidence that they are familiar with nets, proceed to Part Two of the lesson.
2. Provide intervention activities to familiarize students with nets.
   a. Distribute a set of hexomino patterns, Attachment A, Hexomino Patterns, pages one through seven to groups of four students. Discuss the attributes of a cube. Have groups make lists of attributes of a cube (number of sides, all sides are congruent, angle measures, lines of symmetry). Give a sheet of paper to one student in the group. The student writes an attribute that he/she knows and initials the attribute. The student passes the paper to the next person who writes another attribute of a cube and initials. Give two to three minutes to develop the lists. Have a reporter share the groups’ knowledge of cubes. Students should recognize that a cube is made of six identical squares.
   b. Refer students to the hexominoes. Each shape contains six sides that are square. Ask students, “Which shapes could be folded on the lines to create a cube?” Have students classify the hexominoes into nets that make cubes and those that do not. For students needing assistance, allow them to cut a few of the hexominoes and fold them to see if they make cubes. Divide the eight pages of hexominoes among the group members.
   c. Groups share results about which hexominoes are nets and which are not.
   d. Ask students if there are common characteristics of the nets for a cube. Give time for the groups to discuss and share conjectures or counter-examples for nets of cubes. For example, shapes with four blocks making a square and having a common vertex will not be nets.
Another example, shapes with the squares extended on the same side will not be nets.

e. Have the reporters share comparisons developed by the groups. Compose a list of conjectures for nets on chart paper.

f. Introduce the term “net.” Explain that any hexomino which creates a cube when folded is called a net. Provide students time to write a definition of net in their own words in journals or notebooks. Direct the students to complete a “quick-chat” with partners using the following questions:
- What conjectures can we make about nets and non-nets?
- How can we use the conjectures to develop nets for other solids?

Part Two  Conjectures for Nets

3. Tell students to make additional examples of nets and non-nets for geometric objects. Provide each pair of students with a model of a geometric solid (rectangular prism, triangular prism, pyramid with a triangular base, pyramid with a rectangular base, cone and cylinder). Students create two nets and two non-nets (includes all faces of the objects) for the solid.

4. After drawing examples of the nets and non-nets, direct pairs of students to meet with other pairs of students. Students share the example they have created to determine which drawings are nets and which are not. Give two minutes for the partners to share before changing partners.

5. Develop a list of conjectures for the net of each three-dimensional object. See examples in Part One of conjectures for nets of cubes. Examples of conjectures may include:
- The edges of a base of a pyramid have to be adjacent to the edges of the lateral faces;
- Prisms have at least two sets of congruent sides;
- Opposite sides of a pyramid with a rectangular base are congruent (incorrect).

6. Ask students to share their conjectures and record them on the board or chart paper.

7. Tell students to test two conjectures from the list and answer the following questions:
- What evidence do you have to support your conjecture?
- Are there any counter-examples that support or do not support the conjecture?

They record notes and sketches in a journal or notebook.
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8. Have students present the conjectures they tested and what they discovered. Come to consensus on each of the conjectures and keep posted for further inquiry as the lesson proceeds.

9. Closure: Facilitate a class discussion to summarize the concept of nets. Sample questions include:
   - What are nets?
   - What are some attributes that would help you know when a drawing will or will not create a net?
   - Why do we learn about nets? How can they help us? Who might use them?
   - Reflections of What I Know – Journal Writing
     a. Explain nets to an absent friend.
     b. Draw a net on the Mathematics Exit Note, Attachment B, and describe nets in your own words.

Part Three  Finding Surface Area of Pyramids

10. Review previous day’s lesson by showing students pictures of nets and asking them to name the three-dimensional object that each net will create. Show non-nets as well so students make a distinction. Choose examples from student drawings created during the previous lesson.

11. Show students a box. Ask them to describe the surface area of the box. Ask them what they know about finding the surface area of a rectangular prism or box. Allow students to choose partners and discuss what they know about finding surface area. Listen to student discussions to determine if students use formal or informal strategies, understand what surface area is or use strategies for volume. Record this information on an observation checklist or make anecdotal notes.

12. Facilitate a class discussion about how to measure surface area.

13. Show students models of pyramids. Tell them you want to know the least amount of paper needed to cover all sides of the pyramid. Ask students:
   - What can we do to find out how much paper is needed to cover the pyramid?
   - What measure are we looking for to cover the pyramid? (surface area)
   - Can nets be used? How?

   Provide small groups time to develop a plan to figure the surface area of the pyramid.

14. Ask each group to share its plan. As a class, determine what information, strategies and formulas students may use to efficiently find the surface area. Students should conclude the following:
   - The sides of the pyramid have the same area. Ask questions to challenge the conclusion such as:
     a. Does the point of a pyramid always have to lie directly above the center of the base?
     b. How would this effect the lateral faces? Would the faces have the same area?
   - The formulas needed for finding the area of a triangle and of a square. Ask questions to extend to other pyramids such as:
     a. What if the base was a pentagon?
     b. What if the base was an octagon?
   - Add the area of the base.
   - A net shows all of the sides.
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15. Direct students to find the surface area for a pyramid using *Pyramid Net*, Attachment D, and provide additional exercises to find the surface area of pyramids. Ask them to explain how to use parts of the pyramid to find the surface area.

**Instructional Tips:**
Ask students to draw nets of pyramids on grid paper and exchange with other students to practice exercises finding the surface area of pyramids. Allow students to use calculators, rulers and formulas to figure measures. Use software and graphing calculators to draw nets.

16. Facilitate a class discussion to review nets and the surface area of pyramids. Direct the students to complete a *Nets Quick-Write*, Attachment C, about what they have learned about nets and finding surface areas of pyramids, including formulas.

**Part Four  Finding the Surface Area of Cylinders**

17. Show students a cylinder and ask them to sketch a net of the cylinder on grid paper. Have students share sketches in small groups and ensure that all students can sketch a net of a cylinder appropriately.

18. Pose this situation:

   *I want to paint the outside of a decorative pillar that has a height of 48 inches and a diameter of 16 inches. One small canister of paint will cover about 200 square inches. How many small canisters of paint will I need to paint the cylinder?*

19. Present the following questions for small group discussion:
   - What do I need to know about the cylinder? (surface area)
   - Can a net help me find the answer? How?
   - What do I already know about the area of a rectangle? circle?

20. Direct small groups to discuss the questions and develop plans to determine the surface area of a cylinder. Distribute paper canisters or paper tubes that students can cut open and lay flat on grid paper. Tell students to focus on developing formulas or strategies, not on calculating the surface area in the given situation.

21. Bring groups together to discuss strategies for finding the surface area of the cylinder. Ask how nets help them understand the surface area and develop strategies to solve the problem. Summarize strategies and formulas. Students should conclude that they have to:
   - Find the area of a tube by multiplying the circumference of the circle base by the height of the cylinder.
   - Find the area of one of the circle bases.
   - Double the area of the circle for the top and bottom of the cylinder.
   - Add the area of the two circles and the area of the tube surface.

22. Direct students to use their strategies to find the surface area of cylinder objects and models, and solve the problem posed at the beginning of the lesson. Monitor students, observing the use of the strategies and providing assistance, when necessary and appropriate.

23. **Closure:** Direct students to complete a Attachment C, *Nets Quick-Write*, by completing a paragraph which begins with the phrase, “I can show understanding of finding surface area of cylinders...” If time permits, allow students to share with the teacher or other students.
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Differentiated Instructional Support:
Instruction is differentiated according to learner needs, to help all learners either meet the intent of the specified indicator(s) or, if the indicator is already met, to advance beyond the specified indicator(s).

- For sketching the nets of the three-dimensional objects, have two-dimensional shapes available to allow the students to select the appropriate shapes and tape them together. Students lay the two-dimensional shape on the three-dimensional object and then tape the pieces together.
- For students needing additional help, use a dab of rubber cement on each face of the object in order to attach each two-dimensional face to the three-dimensional object. Have students tape a hinge on each edge. Then, cut the hinges one at a time until the net can be laid flat on the desk.
- To differentiate assessment, allow students needing assistance to choose nets from a given set of patterns instead of drawing the nets. Accept oral explanations when students provide reasons why the drawing is or is not a net.
- Challenge students who exceed standard expectations to create sets of nets that are similar, and create prisms and pyramids. Direct the students to find relationships between the surface area and volume of the similar figures. Develop conjectures about the volume and surface areas, relate to the concepts of scale factor and proportional reasoning.

Extensions:
- Creating Similar Nets Activity
  a. Extend the use of nets and provide more opportunities for drawing nets, students create similar nets. They find the relationship between surface area and volume of similar nets and look for patterns to make conjectures about surface area and volume of similar figures.
  b. Ask students what the terms “similar” and “congruent” mean in geometry. Tell them to discuss their knowledge with partners. Ask a few students to report to the class. Summarize similarity and congruence.
  c. Distribute Attachment E, Creating Similar Nets, and extra sheets of grid paper. Students use nets to create similar objects for prisms, pyramids, cones and cylinders.
  d. Ask students to propose methods to create similar nets and to create two nets proportionally similar to the net on the worksheet.
  e. Provide time for discussion of procedures and creation of nets. Observe discussion and provide assistance to groups as needed.
  f. Ask each group to present its nets to another group and discuss.
  g. Direct students to find the surface area and volume for each of the figures. After a few minutes, ask students to analyze surface area and volume measurements. Ask if there are patterns in the measurements that relate to the proportions used to create the net. Make conjectures about surface area, volume and similarity of three-dimensional objects.
  h. Direct students to make similar nets on grid paper for a triangular prism, rectangular pyramid, cone and cylinder.
- Students make a net for an irregular three-dimensional object which has not been used during the lesson. (For example, obelisk, egg-shaped pantyhose carton, a pentagonal or
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hexagonal prism, an icosahedron, octahedron, dodecahedron, truncated cube, truncated octahedron, truncated tetrahedron, truncated icosahedron, the child’s initial in three-dimensional form, etc.

- Ask students what a net for a sphere might look like. Allow students to work in groups to create a net for a sphere and then present their ideas to the class. Allow students to evaluate the ideas of each group constructively, focusing on the attributes of the sphere and proposed net. Show students two-dimensional maps of Earth as an example.

Homework Options and Home Connections:
- Make a net for a cereal box, tissue box, etc…, on grid paper. Determine the surface area and create a net similar to the box. Students working beyond the standard may create nets for irregular objects

Interdisciplinary Connections:
Art  Make mobiles or sculptures with the three-dimensional objects cut and assembled from the nets.

Materials and Resources:
The inclusion of a specific resource in any lesson formulated by the Ohio Department of Education should not be interpreted as an endorsement of that particular resource, or any of its contents, by the Ohio Department of Education. The Ohio Department of Education does not endorse any particular resource. The Web addresses listed are for a given site’s main page, therefore, it may be necessary to search within that site to find the specific information required for a given lesson. Please note that information published on the Internet changes over time, therefore the links provided may no longer contain the specific information related to a given lesson. Teachers are advised to preview all sites before using them with students.

For the teacher:  Hexomino patterns, models of geometric solids, pattern blocks

For the student:  Hexomino patterns, models of geometric solids, grid paper, transparent tape, scissors

Vocabulary:
- attributes
- cone
- conjecture
- cube
- cylinder
- edge
- face
- hexagonal prism
- net
- octagonal prism
- triangular prism
- triangular pyramid
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**Technology Connection:**
- Use computer software or graphing calculators for drawing nets.

**Research Connections:**


**General Tips:**
- Copy hexomino patterns on card stock and laminate to preserve for several years.
- For further development of formulas for volume and surface area, have students make additional nets on cm grid paper and wrap solids made from centimeter cubes.

**Attachments:**
Attachment A, *Hexomino Patterns*
Attachment B, *Mathematics Exit Note*
Attachment C, *Nets Quick Write*
Attachment D, *Pyramid Net*
Attachment E, *Creating Similar Nets*
Attachment A
Hexomino Patterns
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Attachment A (Continued)
What I know about nets...

Evidence of my understanding
Draw examples of four different nets.
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Attachment D
Pyramid Net
Attachment E
Creating Similar Nets

Directions: Draw two nets that are similar to the net below. One should be smaller and one larger. Compare the surface area and volume of the three nets. Analyze the measurements for patterns. Describe the pattern and make a conjecture about similar figures.