**Ohio Standards Connection:**

**Patterns, Functions and Algebra**

**Benchmark I**
Model and solve problem situations involving direct and inverse variation.

**Indicator 14**
Differentiate and explain types of changes in mathematical relationships, such as linear vs. nonlinear, continuous vs. non-continuous, direct variation vs. inverse variation.

**Benchmark B**
Identify and classify functions as linear or nonlinear, and contrast their properties using tables, graphs or equations.

**Indicator 3**
Identify functions as linear or nonlinear based on information given in a table, graph or equation.

**Lesson Summary:**
Students participate in two data collection activities which introduce direct and inverse variation. In the pre-assessment, students categorize numerical, graphical and verbal representations as linear or nonlinear and determine which linear representations are also direct variation relationships. In the post-assessment activity, students convey their understanding of linear, nonlinear, direct and inverse relationships through verbal and written communication. This three-part lesson provides opportunities for students to work individually, with partners and in small groups. During activities, monitor student progress in order to make informal assessments of student understanding and to provide intervention, when necessary.

**Estimated Duration:** Three to four hours

**Commentary:**
The lesson was field tested by teachers across the state of Ohio. Some teacher comments:
- “The students had to use their own learning from the pre-assessment to succeed on the post-assessment.”
- “The post-assessment writing sample gave the students time to reflect on their own learning.”
- “My classes had many conversations about direct variation and the relationship to linear.”
- “We did individual hands-on work, then discussed results with partners and did writing examples and related them to the real world.”

**Pre-Assessment:**
The pre-assessment determines the extent of students’ prior knowledge of linear and nonlinear functions, as well as direct and inverse variations, before any formal review and instruction occur. Students work individually, and then with a partner, to identify numerical, graphical and verbal representations as linear or nonlinear. Following a whole class discussion, students identify those representations which also represent direct variations. Students informally explore the difference between direct and inverse variations.
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Ohio Standards Connection:

Measurement

Benchmark E
Estimate and compute various attributes, including length, angle measure, area, surface area and volume, to a specified level of precision.

Indicator 3
Use appropriate level of precision when calculating with measurements.

Mathematical Processes Benchmarks

D. Apply reasoning processes and skills to construct logical verifications or counter-examples to test conjectures and to justify and defend algorithm solutions.

F. Use precise mathematical language and notations to represent problem situations and mathematical ideas.

G. Write clearly and coherently about mathematical thinking and ideas.

- Distribute Attachment A, Linear vs. Nonlinear Activity Sheet.
- Instruct students to cut out numerical, graphical and verbal representations from the Linear vs. Nonlinear Activity Sheet.
- Instruct students to sort the representations as linear or nonlinear relationships and to place representations in the appropriate sections on the worksheet. Students should complete this activity independently. Monitor student progress.
- Direct students to compare their results with a peer.
- Lead the class in a discussion about the criteria used to sort the representations. (Students may not yet have a formal understanding of slope, but focus their attention on those relationships which have a constant rate of change.)
- Instruct students to finalize their placements and to tape (or glue) each representation onto their papers in the appropriate locations.
- Introduce the characteristics of a direct variation (linear relationship; in the form $y = kx$, where $k$ is the constant of variation; graph passes through the origin.)
- Challenge students to determine which linear representations are direct variations. Instruct students to indicate these on their worksheet.
- Instruct students to list the characteristics of a direct variation in the “Notes” section of the worksheet.
- Facilitate an informal discussion on the numerical difference between direct and inverse variation relationships. For example: When working with tables representing direct variation, such as item 2, the $y$-value increases as the corresponding $x$-value increases. However, when working with an inverse relationship, such as item 3, the $y$-values decrease as the $x$-values increase.
- Use the answer key and scoring guidelines provided to evaluate student work. Identify those students in need of remediation and/or students in need of more challenging questions and activities related to variation.
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Instructional Tip:
Provide pre-cut packets of any and all lesson items (such as the numerical, symbolic, and graphical representations used in the pre-assessment activity) for students with limited fine-muscle control. Consider preparing pre-cut packets for all students. In addition, some students may benefit from the use of guided notes, photocopied notes, or the use of a computer/laptop for the note taking and writing purposes throughout this multi-day lesson.

Linear vs. Nonlinear Answer Key:
- Linear: Items 1, 5, 8
- Linear and Direct: Items 2, 4, 9, 10, 14, 15
- Non-Linear: Items: 3, 6, 7, 11, 12, 13

Scoring Guidelines:
Evaluate student responses using a holistic rubric:
3 Categorizes all models/representations as linear and nonlinear relationships accurately. Recognizes direct and inverse variation relationships.
2 Makes one to three errors in sorting models/representations. Correctly categorizes items in each category.
1 Makes four to seven errors in sorting models/representation. May have difficulty identifying direct variation relationships; e.g., sorts relationship into “linear” category instead of “linear and direct category.”
0 Sorts less than half of the relationships correctly. Demonstrates little understanding of linear and nonlinear relationships.

Post-Assessment:
Use the post-assessment activity, Part Three of the lesson, a review activity, an informal assessment or a formal assessment. The Got Mail? activity is adapted with permission from the Envelope Game created by Siva Thiagarajan.

Students complete the Got Mail? activity, individually or with a partner. Students rotate through four stations, responding thoroughly to an item at each station. Students evaluate the responses created by their peers at the fifth station, selecting the most thorough and accurate response and justifying the selection. Complete, detailed instructions for the post-assessment activity begin at step 17 in the Instructional Procedures for the lesson.

As an optional post-assessment strategy prepare a student hand-out using the five items from the Got Mail? activity. Have students respond to all five questions independently. Evaluate student responses for accuracy.
Scoring Guidelines:
An analytic rubric, Attachment B, Sample Got Mail? Scoring Guidelines can be used to score student responses. Assign weights to the two categories when the activity is used as a formal assessment. The Mathematical Understanding category should weigh more heavily when student responses are assigned a numerical score.

Instructional Procedures:
Part One: X Marks the Spot Activity
1. Facilitate the pre-assessment activity, Linear vs. Nonlinear, using the instructions outlined in the Pre-Assessment section.
2. Facilitate the X Marks the Spot data collection activity, which provides a hands-on introduction to direct variations.
   a. Distribute Attachment C, X Marks the Spot Activity Sheet.
   b. Provide an overview of the activity and directions.
   c. Instruct students to start making “X’s” on a blank sheet of writing/notebook paper after the count of 3 (1-2-3) for five seconds.
   d. Have students count the number of “X’s” made in five seconds.
   e. Instruct students to record his/her individual tally on the table on the X Marks the Spot activity sheet. Have students predict how many “X’s” they can make in 10 seconds and 20 seconds, based on their results for five seconds.
   f. Repeat the data collection procedure for 10, 15, 20, 25 and 30-second intervals.

   Instructional Tip:
   Students with limited fine-muscle control may need to make a mark other than “X” or perform a different task, such as tapping a pencil or pressing a key on a computer keyboard or use an assistive technology tool.

3. Direct student attention to the remaining sections of the activity sheet. Have students plot the results of the data collection on the grid provided and respond to the analysis questions.
   a. Assist students, as needed, in recognizing that the table of values provides data that can be described using ordered pairs: number of seconds, number of “X’s”
   b. Monitor progress and provide assistance as needed when students plot data and begin the analysis. Some students may need guidance in making the graph; e.g., deciding which data represents the independent variable (number of seconds) and deciding what intervals to uses on the axes (how many seconds and how many “X’s” each “tic” mark should represent).

4. Facilitate a discussion about the relationships evident in the data. See sample responses for the questions on Attachment D, X Marks the Spot Answer Key. Bring closure to the activity using summary and probing questions, such as:
   - Why did today’s activity produce data with a linear-like relationship between variables? 
     *It takes approximately the same amount of time to make each “X”. There is an approximate constant rate of change.*
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- Does everyone’s graph look the same? Linear? Students may have used different intervals and students have different skill levels or rates for making the “X’s.” It is likely that all data might not be perfectly linear.
- What factors might have contributed to the data not being perfectly linear? Possible factors include a tired hand and human error in timing.
- Not only does this activity produce linear-like results, it produces a direct variation. Why? The data set contains the point (0,0) -- (0 seconds, 0 “X’s”)

Part Two
5. Prepare the following materials prior to beginning the Teeter-Totter activity:
   a. Make mini-fulcrums or find an appropriate substitute. Directions for making mini-fulcrums:

   ![Diagram of mini-fulcrum]

   Using 4” by 6” cards, cut into 0.5” by 6” strips.
   1. Overlay two strips to create a two-ply strip.
   2. Fold the two-ply strip into fourths.
   3. Overlap the strip’s ends so that a small triangle is formed.
   4. Tape the ends together.
   b. Prepare “Linear,” “Linear and Direct,” and “Nonlinear” signs and post signs in three different areas of the classroom.
   c. Prepare one-per-page versions of items 1-12 on Attachment A, Linear vs. Nonlinear Pre-Assessment Activity Sheet. These may be created by enlarging the tables, equations and graphs on a copy machine on pasting them on a sheet of paper or large note card.
   d. Organize materials for easy distribution.
5. Organize students into collaborative groups of three or four students and give each group one enlarged item.
6. Facilitate the “Linear,” “Linear and Direct,” or “Nonlinear” sign activity.
   a. Instruct students to categorize the item received in step 6 as “Linear,” “Linear and Direct” or “Nonlinear”.
   b. Have one student from each group move to the sign matching the category of the item.
   c. Have students at each sign share or describe the item and why it belongs in the chosen category. Ask the class to verify that the items are correctly identified.
   d. Review the numerical and graphical attributes of a direct variation relationship.
7. Facilitate a group review of student responses to the questions on the X Marks the Spot activity sheet, as needed.
   a. Instruct students to discuss their responses to the analysis questions with their group members. Monitor group discussions.
b. Reassemble the class for a follow-up discussion on items students have questions. Ask students to share the situations that they created for the table in item #8.

9. Inform students that they will be working with a special type of nonlinear relationship during the next activity. Do not inform students that it is with an inverse variation.

10. Facilitate “A Balancing Act” and “Teeter-Totter” activities.
   a. Instruct one student from each group to get the group’s supplies: pre-made mini-fulcrum, ruler, pennies and Attachment E, A Balancing Act Activity Sheet.
   b. Direct students to:
      - Place the fulcrum on a desk so that one “open side” lays flat against the desk, as illustrated on the activity sheet;
      - Balance the ruler on the mini-fulcrum;
      - Place 6 pennies on the 8-inch mark;
      - Place 3 pennies on the 4-inch mark.
   c. Challenge students to move one or both stacks of pennies so that equilibrium is created. Monitor student progress.
   d. Challenge students to come up with several different combinations of pennies and distances that will result in balance situations. Ask students to generalize their results.
   e. Reassemble the class. Ask students to share their generalizations with their classmates.
   f. Allow time for students to summarize/generalize their findings on their A Balancing Act activity sheet.

11. Begin transition to Teeter-Totter activity while students are recording their findings.
    Distribute pre-cut weights from Attachment G, Weights for Teeter-Totter Activity or scissors for students to use and copies of Attachment G, one per student.
    a. Instruct each student to cut out activity sheet weights.
    b. Direct students to place the 50-pound weight at the 3-foot mark. Ask students to determine where a 75-pound weight would need to located in order to balance this teeter-totter. Model steps c and d with objects cut from a transparency made from Attachment G.
    c. Challenge students to create additional combinations that result in a balanced teeter-totter. Instruct students to record possible combinations in the tables (Part Two: Teeter-Totter activity) on their A Balancing Act activity sheet. Monitor student progress. Assist students, as needed. See Attachment F, A Balancing Act Answer Key for sample student responses.
    d. Instruct students to work on Part Three: Length-Width-Area, once they have successfully completed the Teeter-Totter activity. Reassemble students for lesson closure when 10 minutes remain.

12. Reassemble students for lesson closure when 10 minutes remain.
    a. Distribute copies of Attachment G, Parting Thoughts Form.
    b. Instruct students to complete the two tables on the form:
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**Direct Variation**

<table>
<thead>
<tr>
<th>x</th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>?</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>20</td>
<td>40</td>
<td>?</td>
<td>100</td>
</tr>
</tbody>
</table>

**“Teeter-Totter” Variation**

<table>
<thead>
<tr>
<th>x</th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>?</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>24</td>
<td>12</td>
<td>?</td>
<td>8</td>
</tr>
</tbody>
</table>

c. Ask students to suggest another possible name for the second type of variation. Introduce the term “inverse” at this time.
d. Instruct students to write a brief description of the difference between direct and inverse variations on their Attachment H, *Parting Thoughts* form.
e. Collect *Parting Thoughts* forms from each student to serve as a quick, informal assessment. An answer key is provided in Attachment H, *Parting Thoughts Answer Key*.
f. Remind students to finish the *A Balancing Act* activity sheet for homework, focusing on Part 3: Length-Width-Area portion.

### Part Three

13. Organize materials for distribution prior to beginning the *Things I Know to be True* Activity. Prepare sheets of chart paper labeled “Direct Variation” and “Inverse Variation.”
14. Post “Direct Variation” and “Inverse Variation” sheets in the front of the classroom.
15. Facilitate opening vocabulary/terminology review activity.
   a. Instruct students to write three things they remember about direct and inverse variations from Attachment H.
   b. Monitor student work – paying special attention to those students whose response indicates a lack of understanding or may not be able to make connection to concepts in the previous activities.
   c. Consider pairing struggling students with a classmate who would serve as an effective peer tutor during today’s activities.
   d. After three minutes, instruct students to compare their responses with those of a classmate.
   e. Elicit student responses during a whole-class, follow-up discussion. Record responses on “Direct Variation” and “Inverse Variation” sheets
   a. Instruct students to discuss responses with classmates.
   b. Monitor student discussions, answering questions and clarifying, as needed.
   c. Reassemble the class to address items about which a number of students have questions. Address concerns/misconceptions from the *Parting Thoughts* writing activity.
17. Facilitate the *Got Mail?* activity, Attachment B.
   a. Arrange desks or tables in five stations.
   b. Organize students into five groups. Students who need intervention/remediation should be placed in a group that will provide an opportunity for peer tutoring.
   c. Distribute four index cards or half-sheet of blank paper to each student.
   d. Distribute one “*Got Mail?*” packet to each group.
18. Instruct students to work individually, or in pairs, to respond to the problem on the front of the envelope. Each student writes his/her solution on an index card and places his/her index
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card in the envelope. Inform students that supporting work or explanation for the answer needs to be provided. Monitor student work and discussions. After 4-6 minutes, instruct students to leave envelopes at their respective stations.

19. Direct students to move to a second station and solve the problem. Each student writes his/her solution on an index card and places the card in the envelope. (Consider rotating or passing the envelopes from one group to another if available classroom space makes it difficult to physically rotate through stations.) Repeat this procedure two more times until all students have answered four problems.

20. Inform students that they will be completing a different task at the last station. Direct them to evaluate the responses in the envelopes.
   a. Elicit student input in creating a rubric and/or review the scoring guidelines in Attachment B, Sample Post-Assessment Scoring Guidelines. Students may need clarification for the task. They must understand they are to check on the accuracy of and to verify supporting work for each response in the envelope.
   b. Instruct students to move to the fifth station and to remove all index cards and the transparency from that station’s envelope. Ask students, as a group, to review the solutions on the cards and decide which response they believe to be the most thorough and accurate. In addition, students select a group reporter to share the group’s decision with the class. Group reporters should be prepared to defend/explain the group’s selection.
   c. Allot approximately 10-15 minutes for students to complete this task. Monitor student efforts.

21. Call each group’s reporter to the overhead projector to recap the group’s problem and share the solution selected as most thorough and accurate with the class. Schedule presentations for the next class period if using this activity as part of a formal assessment.

22. Ask students to place all cards in the envelopes and collect the envelopes. Review the individual solutions, which may be used as a formal post-assessment. Sort by student and use the Mathematical Understanding component of the analytic rubric to evaluate student progress. Identify any common misconceptions and students who may need additional instruction or intervention.

Differentiated Instructional Support:
Instruction is differentiated according to learner needs, to help all learners either meet the intent of the specified indicator(s) or, if the indicator is already met, to advance beyond the specified indicator(s).
   • Provide pre-cut packets of lesson materials for students with limited fine-muscle control.
   • Provide guided notes, photocopied notes or access to a computer/electronic device for note taking and writing throughout the lesson.
   • Use a piece of uncooked spaghetti to place a line of best fit for the plotted data points. Have students write an equation that approximates the line of best fit. This provides an opportunity to discuss slope as a rate of change.

Extension:
• Use graphics calculators, calculator-based data collection devices, and pressure sensor probes to investigate the inverse relationship between volume and pressure known as Boyle’s Law.
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Home Connections:
- Encourage students to discuss the characteristics of linear, nonlinear, direct, and inverse relationships with parents and guardians. In addition, students, parents, and guardians can look for real-life examples of these relationships at home.
- Making popcorn can serve as a model of an inverse relationship. Students could make microwave popcorn for a family movie night. During this activity, students could point out the fact that as the kernels pop, the popcorn’s volume increases, which means the “empty space” in the bag decreases.

Materials and Resources:
The inclusion of a specific resource in any lesson formulated by the Ohio Department of Education should not be interpreted as an endorsement of that particular resource, or any of its contents, by the Ohio Department of Education. The Ohio Department of Education does not endorse any particular resource. The Web addresses listed are for a given site’s main page, therefore, it may be necessary to search within that site to find the specific information required for a given lesson. Please note that information published on the Internet changes over time, therefore the links provided may no longer contain the specific information related to a given lesson. Teachers are advised to preview all sites before using them with students.

Pre-Assessment:
For the teacher: Individual copies of items 1-12 on Attachment A, Linear vs. Nonlinear Pre-assessment on transparencies or presentation software.
For the student: Scissors, glue stick or tape, copies of Attachment A for each student.

Part One:
For the teacher: Stopwatch or clock with second hand.
For the student: Copies of Attachment C for each student.

Part Two:
For the teacher: Note cards to make mini-fulcrum (one for each group of 3-4 students) or appropriate substitute, such as fulcrum borrowed from a science classroom; three signs labeled “Linear,” “Linear and Direct” and “Nonlinear;” one-per-page versions of items 1-12 on Attachment A; copy of Attachment , Teeter-Totter Activity Sheet on transparency or projection software
For each student: Copies of Attachments E, G and H.
For each group of students: Index Card Mini-Fulcrum; ruler; nine pennies

Part Three:
For the teacher: Chart paper, envelopes with problems attached
For the student: Four note cards or half-sheets of paper per student, copies of Attachments B (optional) and I.
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**Vocabulary:**
- direct variation
- inverse variation
- linear
- nonlinear
- rate of change

**Technology Connections:**

**Part One:**
- Use a graphing calculator and enter data into stat lists. Select stat plot settings that produce a scatter plot, and adjust window settings to view all data points. Students enter linear equations until they find one that models the data. Use spreadsheets or graphing software programs to investigate data for the data collection activity.

**Part Two:**
- Use a graphics calculator when completed the Part Three: Length-Width-Area on the A Balancing Act activity sheet. Enter data into stat lists, select Stat Plot settings that produce a scatter plot, and adjust window settings to view all data points. Students then enter inverse variation equations until they find one that models the data ($xy = 24$). This would enable students to see both branches of the curve – not just the first quadrant branch. Use spreadsheets or graphing software programs to investigate the data.

**Part Three:**
- Enter a direct or inverse equation for $y$ in a graphing calculator or in graphing software. Set a table of values by entering starting and incremental values for the independent variable, $x$. Use a projection device to display the table set’s starting and incremental values. Challenge students to type an equation into their graphing calculators that will produce the same table of values.

**Research Connections:**


**Attachments:**
- Attachment A, Linear vs. Nonlinear Pre-Assessment and Worksheets
- Attachment B, Sample Got Mail? Sample Items and Scoring Guidelines
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Attachment C, X Marks the Spot Activity Sheet
Attachment D, X Marks the Spot Answer Key
Attachment E, A Balancing Act Activity Sheet
Attachment F, A Balancing Act Answer Key
Attachment G, Weights for Teeter-Totter Activity
Attachment H, Parting Thoughts Form
Attachment I, Things I Know to Be True Form
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Attachment A
Linear vs. Nonlinear Pre-Assessment

- Cut out the following representations.
- Determine whether each relationship represented by the tables, equations, graphs and written descriptions is linear or nonlinear.
- Place each item in the appropriate column on your “Linear vs. Nonlinear Worksheet”

1. \[ y = 2x \]
2. \[ y = 2x + 5 \]
3. \[ xy = 20 \]
4. \[ y = x^2 \]
5. \[ y = 2x + 5 \]
6. \[ xy = 20 \]
7. \[ y = x^2 \]
8. [Graph of a straight line]
9. [Graph of a straight line]
10. [Graph of a straight line]
11. [Graph of a parabola]
12. [Graph of a parabola]

13. The length and width of rectangles with areas of 20 square units
14. The number of hours and the distance traveled when driving in a car at a constant rate of 60 mph
15. The number of touchdowns scored in a football game and the points scored (from touchdowns)
Name ________________________________  Date ____________

Linear

Notes:
Nonlinear
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Attachment B
“Got Mail” Possible Review Items/Questions

1. Explain why the “X” Marks the Spot activity represents a direct variation. Explain how the activity could be changed so that it no longer results in a direct variation relationship.

2. Describe a situation, other than “X” Marks the Spot that represents a direct variation relationship.

3. Describe a situation, other than “Teeter-Totter” that represents an inverse variation.

4. Complete the table so that it models a direct variation relationship and write an equation that produces the table’s values.

<table>
<thead>
<tr>
<th>$x$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>40</td>
<td></td>
<td>120</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

5. Create a second table that represents an inverse variation relationship.

6. Draw a graph that represents a direct variation.

7. Draw a second graph that is linear, but does not represent a direct variation.

   Describe the key difference between the two graphs.

   Use the provided illustration as a template to create a graphic organizer for direct variation.
### Sample for “Got Mail” Scoring Guidelines

<table>
<thead>
<tr>
<th></th>
<th>Presentation/Collaboration</th>
<th>Mathematical Understanding</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Expert</strong></td>
<td>• Tasks evenly distributed among all members of group.</td>
<td>• Demonstrates understanding of direct and inverse relationships using examples, accurate descriptions and comparison.</td>
</tr>
<tr>
<td></td>
<td>• Multiple ways of presenting concept (models, visual display, oral discussion).</td>
<td>• Identifies representations as linear or inverse with appropriate and accurate reasoning.</td>
</tr>
<tr>
<td></td>
<td>• Discussions engaging and involve all members of the group.</td>
<td>• Answers questions accurately and provides appropriate support.</td>
</tr>
<tr>
<td></td>
<td>• Uses appropriate words when giving feedback and solves problems through consensus.</td>
<td>• Uses mathematical terms appropriately.</td>
</tr>
<tr>
<td><strong>Proficient</strong></td>
<td>• All members participate, though tasks not equally distributed.</td>
<td>• Demonstrates understanding of direct and inverse relationships using examples and accurate descriptions.</td>
</tr>
<tr>
<td></td>
<td>• Concept presented using two representations (model or visual and oral discussion).</td>
<td>• Identifies representations as linear or inverse with appropriate reasoning.</td>
</tr>
<tr>
<td></td>
<td>• Discussion involves all members of the group.</td>
<td>• Answers questions and provides appropriate support.</td>
</tr>
<tr>
<td></td>
<td>• Uses appropriate words when giving feedback, attempts to solve problems through consensus.</td>
<td>• Uses mathematical terms and common vocabulary appropriately.</td>
</tr>
<tr>
<td><strong>Advanced Beginner</strong></td>
<td>• Two to three members participate in presentation.</td>
<td>• Demonstrates understanding of direct or inverse relationships using examples and descriptions.</td>
</tr>
<tr>
<td></td>
<td>• Concept presented using one representation.</td>
<td>• Identifies representations as linear or inverse with one to two errors.</td>
</tr>
<tr>
<td></td>
<td>• Discussion limited and involves two to three members.</td>
<td>• Answers questions and attempts to provide appropriate support.</td>
</tr>
<tr>
<td></td>
<td>• Uses some appropriate words when giving feedback, solves problems with limited teacher assistance.</td>
<td>• Uses common vocabulary appropriately, limited mathematical terms are used.</td>
</tr>
<tr>
<td><strong>Novice</strong></td>
<td>• One to two members participate in presentation.</td>
<td>• Demonstrates little understanding of direct or inverse relationships</td>
</tr>
<tr>
<td></td>
<td>• Concept presented using one representation.</td>
<td>• Consistently misidentifies representations as linear or inverse.</td>
</tr>
<tr>
<td></td>
<td>• Discussion limited and involves one to two members.</td>
<td>• Answers questions incorrectly and with inappropriate support.</td>
</tr>
<tr>
<td></td>
<td>• Uses some appropriate words when giving feedback, teacher solves problems.</td>
<td>• Uses common vocabulary, however unclear communication of mathematical understanding.</td>
</tr>
</tbody>
</table>
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Attachment C
X Marks the Spot Activity Sheet

Name: _______________________________________ Date: _____________

Group Members _________________________________________________________

Record the number of X’s made during each time period in the table and plot the data points.

1. Record data in the table below:  2. Plot the data points on the grid below:

<table>
<thead>
<tr>
<th>Number of Seconds</th>
<th>Number of “X’s”</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td></td>
</tr>
<tr>
<td>25</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td></td>
</tr>
</tbody>
</table>

Analysis:

3. Describe the relationship between the number of seconds and the number of X’s.

4. Explain why it may be expected that this data collection activity to produce linear-like results.

5. How could the data collection process be changed so that non-linear results would be obtained?
6. Explain why the activity produced a direct variation relationship.

7. How could the data collection process be changed so that the results would still be linear, but not a direct variation?

8. Complete the following table of values.

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>0</td>
<td>5</td>
<td>10</td>
<td>20</td>
</tr>
</tbody>
</table>

Explain why the table represents a direct variation.

9. Describe a situation that could be represented by the table in question 8.
Record the number of X’s made during each time period in the table and plot the data points.

1. Record data in the table below:  
2. Plot the data points on the grid below:

<table>
<thead>
<tr>
<th>Number of Seconds</th>
<th>Number of “X’s”</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td></td>
</tr>
<tr>
<td>25</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td></td>
</tr>
</tbody>
</table>

*Answers will vary. Both should reflect linear-like relationships.*

Analysis:

3. Describe the relationship between the number of seconds and the number of X’s.

   *The number of X’s increases as the number of seconds increases. Some students may describe the relationship using a specific rate; e.g., can draw 20 X’s for each 10 seconds.*

4. Explain why it may be expected that this data collection activity to produce linear-like results.

   *It takes approximately the same amount of time to make each X. There is an approximate constant rate of change.*
Direct and Inverse Variation – Grade Eight

Attachment D (continued)
X Marks the Spot Answer Key

5. How could the data collection process be changed so that non-linear results would be obtained?

*Answers may vary. Students could make or draw a different object during the time periods; e.g., draw Xs during the 5-second time period, dots during the 10-second time period, and stick figures during the 20-second time period.*

6. Explain why the activity produced a direct variation relationship.

*The data includes the point (0,0) – (0 seconds, 0 X’s)*

7. How could the data collection process be changed so that the results would still be linear, but not a direct variation?

*Answers may vary. The students could draw 10 X’s before the timing begins for each trial.*

8. Complete the following table of values.

<table>
<thead>
<tr>
<th></th>
<th>x</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5*</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>0</td>
<td>5</td>
<td>10</td>
<td>15</td>
<td>20</td>
<td>25</td>
<td></td>
</tr>
</tbody>
</table>

*Answers may vary.*

Explain why the table represents a direct variation.

*The table is a direct variation because it is linear (has a constant rate of change of 5) and passes through the origin (0,0).*

9. Describe a situation that could be represented by the table in question 8.

*Answers will vary.*
Part One: Ruler/Penny Activity

- Place the fulcrum on a desk as shown below.
- Balance your ruler on the fulcrum.
- Place a stack of 6 pennies on the 8” mark.
- Place a stack of 3 pennies on 4” mark.

1. Describe at least one move that would balance this “teeter-totter”

2. What generalization can be made based on these moves?

Part Two: Teeter-Totter Activity

3. Record combinations of weights and distances that would balance the illustrated teeter-totter.

<table>
<thead>
<tr>
<th>Weight 1</th>
<th>Weight 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>50 lbs.</td>
<td>75 lbs.</td>
</tr>
<tr>
<td>3 feet from fulcrum</td>
<td>feet from fulcrum</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Part Three: Length-Width-Area

4. A rectangle has an area of 24 square units. Represent all possible rectangles with integral dimensions on the grid below.
5. Record the dimensions for all the rectangles in the table:

<table>
<thead>
<tr>
<th>Length</th>
<th>Width</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

6. Describe the relationship between the rectangle’s length and width values.

Determine whether the table of values represents a direct variation, an inverse variation, or neither of these two relationships.

7. Write an equation that models the table of values. Use $x$ for length and $y$ for width.
1. Describe at least one move that would balance this “teeter-totter”

*Answers will vary. Moving the three-penny stack to the left or moving the six-penny stack to the right will balance the teeter-totter.*

2. What generalization can be made based on these moves?

*Answers will vary. Students should recognize that the product of the mass and distance on one side should be equal to the product of the mass and distance on the other side.*

3. Record combinations of weights and distances that would balance the illustrated teeter-totter.

<table>
<thead>
<tr>
<th>Weight 1</th>
<th>Weight 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>50 lbs.</td>
<td>75 lbs.</td>
</tr>
<tr>
<td>3 feet from fulcrum</td>
<td>2 feet from fulcrum</td>
</tr>
</tbody>
</table>

*Additional answers will vary; however, the product of the weight and distance for Weight 1 should equal that for Weight 2.*

4. A rectangle has an area of 24 square units. Represent all possible rectangles with integral dimensions on the grid below.

*Students should represent the different rectangles. Some students may draw the rectangles on the grid. Others may represent the rectangles as ordered pairs. Some may show a 4-by-6 rectangle and a 6-by-4 rectangle on the graph, and others may not consider that these are different rectangles.*

5. Record the dimensions for all rectangles in the table:

<table>
<thead>
<tr>
<th>Length</th>
<th>Width</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>24</td>
</tr>
<tr>
<td>2</td>
<td>12</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>6</td>
<td>4</td>
</tr>
<tr>
<td>8</td>
<td>3</td>
</tr>
<tr>
<td>12</td>
<td>2</td>
</tr>
<tr>
<td>24</td>
<td>1</td>
</tr>
</tbody>
</table>

*Example responses shown.*
6. Describe the relationship between the rectangle’s length and width values.

*Students descriptions will vary. Students should recognize that as the width decreases as the length increases.*

Determine whether the table of values represents a direct variation, an inverse variation, or neither of these two relationships.

*Inverse variation*

7. Write an equation that models the table of values. Use $x$ for length and $y$ for width.

$$xy = 24$$
Direct and Inverse Variation – Grade Eight

Attachment G
Weights for Teeter-Totter Activity

- Cut out the “weights” from the bottom of this paper.
- Place the 50-pound weight at the teeter-totter’s 3-foot mark.
- Determine where the 75-pound weight would have to sit in order to balance the teeter-totter.
- Find other weight and distance combinations that also balance the teeter-totter. Record these combinations on your “A Balancing Act” Activity Sheet.
Direct and Inverse Variation – Grade Eight

Attachment H
Parting Thoughts: Direct and Inverse Variations

Name ________________________________ Date _________

1. Complete the following tables:

<table>
<thead>
<tr>
<th>Direct Variation</th>
<th>“Teeter-Totter” Variation</th>
</tr>
</thead>
<tbody>
<tr>
<td>x 1 2 4 ?</td>
<td>x 1 2 4 ?</td>
</tr>
<tr>
<td>y 20 40 ? 100</td>
<td>y 24 12 ? 8</td>
</tr>
</tbody>
</table>

2. “Teeter-Totter” Variation is also known as ________________________________

3. Describe the difference(s) between direct and inverse variations.

Parting Thoughts: Direct & Inverse Variations

Name ________________________________ Date _________

1. Complete the following tables:

<table>
<thead>
<tr>
<th>Direct Variation</th>
<th>“Teeter-Totter” Variation</th>
</tr>
</thead>
<tbody>
<tr>
<td>x 1 2 4 ?</td>
<td>x 1 2 4 ?</td>
</tr>
<tr>
<td>y 20 40 ? 100</td>
<td>y 24 12 ? 8</td>
</tr>
</tbody>
</table>

2. “Teeter-Totter” Variation is also known as ________________________________

3. Describe the difference(s) between direct and inverse variations.
1. **Direct Variation**

\[
\begin{array}{c|cccc}
\text{x} & 1 & 2 & 4 & 5 \\
\text{y} & 20 & 40 & 80 & 100 \\
\end{array}
\]

**“Teeter-Totter” Variation**

\[
\begin{array}{c|cccc}
\text{x} & 1 & 2 & 4 & 3 \\
\text{y} & 24 & 12 & 6 & 8 \\
\end{array}
\]

2. “Teeter-Totter” Variation is also known as an inverse variation.

3. Answers Will Vary. Important concepts: Direct variations are linear and include the point (0, 0). Inverse variations are not linear. As one variable increases, the second variable decreases. Some students may also recognize that direct variations are represented by the rule \( y = kx \) or \( \frac{y}{x} = k \), where \( k \) is the constant of variation. Inverse variations are represented by the rule \( xy = k \).
Direct and Inverse Variation – Grade Eight

Attachment I
Things I Know to Be True Form

Name ___________________________________________ Date __________

These Things I Know to be True… about Direct Variations

________________________________________________________________________
________________________________________________________________________
________________________________________________________________________
________________________________________________________________________
________________________________________________________________________
________________________________________________________________________

These Things I Know to be True… about Inverse Variations

________________________________________________________________________
________________________________________________________________________
________________________________________________________________________
________________________________________________________________________
________________________________________________________________________
________________________________________________________________________