### Ohio Standards Connection

**Number, Number Sense and Operations**

**Benchmark D**
Use models, points of reference and equivalent forms of commonly used fractions to judge the size of fractions and to compare, describe and order them.

**Indicator 5**
Use models and points of reference to compare commonly used fractions.

**Benchmark J**
Estimate the results of whole number computations using a variety of strategies, and judge the reasonableness.

**Indicator 9**
Estimate the results of computations involving whole numbers, fractions and decimals, using a variety of strategies.

### Mathematical Processes

**Benchmarks**

G. Use reasoning skills to determine and explain the reasonableness of a solution with respect to the problem situation.

H. Recognize basic valid and invalid arguments, and use examples and counter examples, models, number relationships, and logic to support or refute.

### Lesson Summary:

*In this two-day lesson, students estimate answers to computations that involve fractions. Students use whole numbers as points of reference to compare commonly used fractions. They compare fractions to halves and whole numbers. Students use these skills to estimate the sum or difference of two fractions. Students determine the reasonableness of the estimates.*

**Estimated Duration:** Two hours

### Commentary:

Students should have prior experience working with fractions as a part of a whole or as part of a set in order to succeed with this lesson. Classroom instruction should build on these previous experiences and clarify students’ conceptual understanding and develop language related to fractions. Students should explore fractions concretely through real-world contexts (Burns, 1992). Using area models, students can compare and order fractions using benchmarks zero, one-half and one (NCTM, 2001).

### Pre-Assessment:

**Option 1**
- Assess students’ prior knowledge of fractions by having them identify various fractions and providing examples of those fractions in the real-world.
- Distribute three feet of yarn, eight paper clips and *Comparing Fractions Number Cards*, Attachment A, to each student or pairs of students. Have students measure and cut the yarn or pre-cut the yarn before the lesson. Inform students that the yarn represents a number line. Direct students to cut out the fraction number cards and have them place the fractions in order from least to greatest on the yarn. The paper clips will hold the number cards on the yarn.

**Option 2**
- Distribute Attachment B, *Comparing Fractions Pre-Assessment* to students.
- Have them complete the tasks on the attachment.

### Scoring Guidelines:

Use the appropriate rubrics on *Pre-Assessment Performance Rubrics*, Attachment C, to identify student performance level and inform need for intervention.
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Post-Assessment:
- The post-assessment involves problem solving with short answer responses and single-response items
- Distribute Attachment C, Comparing Fractions Post-Assessment to students. Students should complete the post-assessment independently.

Scoring Guidelines:
Use item-specific rubrics Post-Assessment Scoring Rubrics and Answer Key, Attachment E.

Instructional Procedures:
1. Ask students questions about experiences they have had with fractions such as:
   - When have you used or seen fractions?
   - What have you learned about fractions?
   - What does it mean to have one-half of something?
   - Is one-half of a small pizza different than one-half of a large pizza?
Write responses on the board or chart paper. Responses will vary. Some students may refer to one-half as a part of a set. For example, there are four chocolates in a pack. I ate two of them, so I ate half of the chocolates. Others may refer to one-half as a part of a whole. I have a cake divided into two equal parts. If I eat one of those parts, that is half of the whole cake.
2. Ask students what it means to have one whole of something, and write responses on the board or chart paper. A whole may be seen as all of the parts in a set or an entire part of some object. If I have a bag of 20 cookies, the whole is the total number of cookies in the bag, which is 20. If I have a whole cake sliced into 8 pieces and have all of the 8 pieces, the whole is \( \frac{8}{8} \).
3. Discuss the fraction three-fourths and ask what it represents. For example:
   - If I have three-fourths of a pizza, how much pizza will I need to have 1 whole pizza? (one-fourth)
   - How do you know you only need one-fourth more? (Because \( \frac{3}{4} + \frac{1}{4} = \frac{4}{4} \), and if I have a whole pizza divided into 4 equal parts and I have all 4 of those parts, then I have a whole pizza.)
4. Have students make a model of the fraction \( \frac{1}{4} \). They can draw a picture or show a set of objects with three-fourths of them circled or grouped (Four pattern blocks with three of them red and one of them blue, an item divided into four equal parts with three of them shaded in, etc.) Check student responses.

Instructional Tip:
Provide students physical and visual models of fractions to complete the next step, using points of reference. Fraction circles or bars, fraction rods or fraction cards with visual models can be used to determine what whole number of a given fraction is closer.
5. Draw two columns on the board or chart paper. Label one side “Fractions closer to \( \frac{1}{2} \)” and label the other side “Fractions closer to 1”. Divide students into pairs. Give each pair of students five index cards with these fractions on them: \( \frac{5}{6}, \frac{1}{3}, \frac{1}{4}, \frac{10}{12}, \frac{4}{10} \).
6. Tell students to work with their partners to determine if the given fraction is closer to one-half or one. Have them sort the index cards into the titled groups.
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7. Observe pairs of students and generate questions that ask them to explain how the fractions are sorted. Possible questions and responses include
   - How do you know \( \frac{3}{10} \) is closer to \( \frac{1}{2} \) than 1? (Half of 10 is 5. In order to have a half of 10, I would need \( \frac{5}{10} \). I only have \( \frac{4}{10} \), so I have less than half of 10.) Another response is, I have \( \frac{4}{10} \). I need one more tenth to have \( \frac{5}{10} \), which is half of 10, but I need \( \frac{10}{10} \) which is 1 whole.)

   Clarify student misconceptions and model the problem when needed.

8. Select pairs of students to come to the board or chart and place their fraction index cards in the correct column. Use magnetic strips to stick the index cards on the board or have the students write the fraction on the board. Ask other students if they agree with the placement of the fraction in the correct column on the board. If they agree, they can give a “thumbs up”, and if they disagree, a “thumbs down”. If students disagree with the placement of the fraction, discuss which column they think the fraction should be placed and why.

   Index cards are correctly sorted in this placement.

   \[
   \begin{array}{c|c|c}
   \text{Fractions closer to } \frac{1}{2} & \text{Fractions closer to 1} \\
   \hline
   \frac{1}{3} & \frac{10}{12} & \frac{5}{6} \\
   \frac{1}{4} & \frac{4}{10} & \\
   \frac{1}{5} & \\
   \frac{1}{2} & \end{array}
   \]

9. Add a third column to the two columns on the board and label “Fractions closer to zero.”

10. Pass out two more index cards with \( \frac{2}{12} \) and \( \frac{4}{7} \) to each pair of students.

11. Clear the board of all index cards or fractions. Direct the students to re-sort the index cards using the three columns, Fractions closer to 0, \( \frac{1}{2} \) and 1.

12. Pose the question to students,

   \textit{What column do you think } \frac{3}{4} \textit{ should go in?}

   Some students may say that one-fourth is closer to zero while other students will respond that one-fourth is closer to one-half. The fraction one-fourth is equally close to zero as it is to one-half. If students have had previous experience with rounding, you can review the rule for rounding whole numbers. Have the students come to consensus about where to place one-fourth. Place \( \frac{3}{4} \) in the “Closer to \( \frac{1}{2} \)” column instead of the “Closer to 0” column to be consistent with the rounding rule.

13. Select students to place their index cards in the appropriate column on the board. Ask other students if they agree with the placement of the fraction in the correct column on the board. If they agree they can show “thumbs up”. If they disagree, “thumbs down”.

14. If students disagree with the placement of the fraction, ask them to discuss which column the fraction should be placed in and why. Correct placement is shown.

   \[
   \begin{array}{c|c|c|c}
   \text{Closer to 0} & \text{Closer to } \frac{1}{2} & \text{Closer to 1} \\
   \frac{2}{12} & \frac{1}{4}, \frac{1}{3}, \frac{4}{10} & \frac{5}{6}, \frac{10}{12}, \frac{4}{5} \\
   \end{array}
   \]

Part 2

15. On the board make a two column chart labeled Closer to 1 and Closer to 2. Give the students index cards with the fractions \( \frac{2}{8}, \frac{1}{4}, \frac{1}{7}, \frac{5}{7}, \frac{6}{5}, \) and \( \frac{4}{3} \). Have partners determine if the fractions are closer to one or two, sort the cards into two groups and explain why they sorted them the way they did. For example:
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\[ \frac{4}{3} \] means I have 1 whole and \[ \frac{1}{3} \]. This means I have \( 1\frac{1}{3} \). 1\( \frac{1}{3} \) is closer to one because I need two more thirds to have \( \frac{2}{3} \) and \( 1\frac{1}{3} + \frac{2}{3} = 1\frac{1}{3} = 1 + 1 = 2. \]

16. Select pairs of students to come to the board or chart and place their fraction index cards in the correct column. Ask the other students if they agree with the placement of the fraction, and if they disagree, have them discuss in which column they think the fraction should be placed. The index cards should be sorted in this order:

<table>
<thead>
<tr>
<th>Closer to 1</th>
<th>Closer to 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{2}{8} )</td>
<td>( \frac{3}{4} )</td>
</tr>
<tr>
<td>( \frac{4}{5} )</td>
<td>( \frac{5}{4} )</td>
</tr>
</tbody>
</table>

17. Present additional fraction cards, for example, \( \frac{4}{5} \) and \( \frac{5}{3} \). Have the students show “thumbs up” or “thumbs down” if they think \( \frac{4}{5} \) is closer to one. Have them show “thumbs up” or “thumbs down” if they think \( \frac{5}{3} \) is closer to two. Repeat this process as needed using a variety of common fractions.

18. Add a column labeled “Closer to \( 1\frac{1}{2} \)”. Ask students which fractions are closer to \( 1\frac{1}{2} \) than one or two. Allow partners to determine the fractions. Select students to share fractions and explanations with the class.

19. Pose the problem situation to students. Have students discuss in pairs and share with the class. Discuss the importance of interpreting estimations depending on the situation.

Stella was making a bracelet for her friend Mindy. She measured Mindy’s wrist. Her wrist was five and three-eighths inches around. About how long is Mindy’s wrist? Would this estimate be good to use to make the bracelet? Why or why not? What length of measure may be more appropriate?

20. Provide problem situations in which students estimate fractions.

- Grandma needed two-thirds cup of milk for the pancake recipe and seven-eighths cup of milk for the waffle recipe. About how much milk did grandma need to make pancakes and waffles?
- Caleb measured two worms. One worm measured three and one-eighth inches and the other measured four and three-fourths inches. About how much longer is the second worm?

21. To close the lesson, review math terminology with students: improper fractions, mixed number, and proper fractions. In their mathematics journals, have them write about what they learned about estimating fractions to one-half and whole numbers.

Differentiated Instructional Support:
Instruction is differentiated according to learner needs, to help all learners either meet the intent of the specified indicator(s) or, if the indicator is already met, to advance beyond the specified indicator(s).

- Using physical models and visual representations address needs of the visual learner. Having students discuss ideas related to comparing fractions to one-half and whole numbers benefit the auditory learners. Allowing students to move around the room to discuss with other pairs or groups addresses needs of kinesthetic learners.
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- Use number lines and models of fractions to solve problem situations for students who need additional assistance. Use contexts that are familiar to them and in which they can relate.

**Extension:**
Play the Fraction Classroom Feud using questions created by the students. Collect the questions from the students and check for accuracy.

**Home Connection:**
Give students questions that involve fractions of a set, fractions as part of a whole and improper fractions.

**Materials and Resources:**
The inclusion of a specific resource in any lesson formulated by the Ohio Department of Education should not be interpreted as an endorsement of that particular resource, or any of its contents, by the Ohio Department of Education. The Ohio Department of Education does not endorse any particular resource. The Web addresses listed are for a given site’s main page, therefore, it may be necessary to search within that site to find the specific information required for a given lesson. Please note that information published on the Internet changes over time, therefore the links provided may no longer contain the specific information related to a given lesson. Teachers are advised to preview all sites before using them with students.

For the teacher: chart paper, index cards

For the students: fraction model manipulatives

**Vocabulary:**
- improper fraction
- point of reference
- proper fraction

**Technology Connection:**
Utilize free internet sites that provide skill drills for fractions.

**Research Connections:**


Comparing and Estimating Fractions – Grade Four

Smutny, Joan F. *Differentiation and Creativity (Content)*. A Professional Development Offering of Phi Delta Kappa International. 5 November 2003. Presentation materials.

**General Tips:**
- Prepare index cards with fractions before instruction. Laminate to protect and save the cards.
- Prepare sets of fraction manipulatives for students to use throughout the lesson. Provide these materials for students who need additional assistance.

**Attachments:**
Attachment A, *Comparing Fractions Number Line Activity*
Attachment B, *Comparing Fractions Pre-Assessment*
Attachment C, *Pre-Assessment Performance Rubrics*
Attachment D, *Comparing Fractions Post-Assessment*
Attachment E, *Post-Assessment Performance Rubrics*
### Attachment A
#### Number Line Activity

<table>
<thead>
<tr>
<th>Fraction 1</th>
<th>Fraction 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/4</td>
<td>1 2/3</td>
</tr>
<tr>
<td>5/6</td>
<td>6/6</td>
</tr>
<tr>
<td>4/3</td>
<td>2 1/2</td>
</tr>
<tr>
<td>1/8</td>
<td>1/2</td>
</tr>
</tbody>
</table>
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Attachment B
Pre-Assessment

Name_________________________________________ Date____________________

Directions: Read the problem situations and solve.

1. Jonathan ate $\frac{2}{3}$ of a candy bar while his friend Sonny ate $\frac{1}{4}$ of a candy bar. Who had the most candy left? Explain your answer.

2. Compare using $<$, $>$ or $=$
   
   a. $\frac{1}{2}$ \hspace{5mm} \hspace{5mm} $\frac{2}{4}$
   
   b. $\frac{5}{6}$ \hspace{5mm} $\frac{3}{2}$
   
   c. $\frac{5}{10}$ \hspace{5mm} $\frac{3}{12}$

2. Place the following fractions in order on the number line.

$$\frac{1}{2} \hspace{5mm} \frac{1}{4} \hspace{5mm} \frac{3}{4} \hspace{5mm} \frac{4}{4} \hspace{5mm} \frac{2}{3}$$

4. Place the following fractions in the appropriate column

$$\frac{4}{5} \hspace{5mm} \frac{1}{4} \hspace{5mm} \frac{3}{8} \hspace{5mm} \frac{8}{10}$$

   closer to $1$ \hspace{2cm} closer to $\frac{1}{2}$ \hspace{2cm} closer to $0$
## Attachment C
### Pre-Assessment Performance Rubrics

**Option 1**

<table>
<thead>
<tr>
<th>Performance Descriptors</th>
<th>Checklist</th>
<th>Students in Performance Category</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ordered fractions, including improper fractions, correctly on number line</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ordered fractions properly but had difficulty with improper fractions and mixed numerals</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Had difficulty placing fractions and mixed numbers in order on the number line</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### Attachment C (continued)

#### Pre-Assessment Performance Rubrics

**Option 2**

### Question 1

<table>
<thead>
<tr>
<th>Performance Level Descriptors and Intervention</th>
<th>Adequate Understanding for Instruction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concludes Jonathan has more candy left and provides an accurate explanation. For Example: $\frac{2}{3}$ is less than $\frac{3}{4}$, the person who ate the least amount of candy will have the most left.</td>
<td></td>
</tr>
<tr>
<td>Concludes Jonathan ate the least OR has more candy left, but omits or includes an inaccurate explanation. OR Concludes Sonny ate more of the bar with an inaccurate explanation.</td>
<td></td>
</tr>
<tr>
<td>Focus on developing explanations through discourse and writing strategies.</td>
<td></td>
</tr>
<tr>
<td>Inadequate Understanding</td>
<td>Concludes Sonny has more of the candy bar left or that Jonathan ate more than Sonny.</td>
</tr>
<tr>
<td>Provide additional instruction of fraction size and comparing sets of fractions using physical models and visual representations.</td>
<td></td>
</tr>
</tbody>
</table>

**Question 2**

- a. =
- b. <
- c. >

**Question 3**

<table>
<thead>
<tr>
<th>Performance Level Descriptors and Intervention</th>
<th>Adequate Understanding for Instruction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Places all of the fractions in order from least to greatest on the number line. For example: $\frac{1}{4}, \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{4}$</td>
<td></td>
</tr>
<tr>
<td>One of the numbers incorrectly placed on the number line.</td>
<td></td>
</tr>
<tr>
<td>Two or more numbers incorrectly placed on the number.</td>
<td>Focus intervention on size of fraction using physical models and visual representations.</td>
</tr>
</tbody>
</table>
### Question 4 Performance Level Descriptors and Intervention

<table>
<thead>
<tr>
<th>Adequate Understanding for Instruction Beyond Expectations</th>
<th>Performance Level Descriptors and Intervention</th>
</tr>
</thead>
<tbody>
<tr>
<td>Places all 3-4 fractions in the correct columns. For example:</td>
<td>$\frac{1}{2}$ closer to 1, $\frac{8}{10}$ closer to $\frac{1}{2}$, $\frac{3}{8}$ closer to 0, $\frac{1}{8}$ closer to 0</td>
</tr>
<tr>
<td>Instruction is needed.</td>
<td>Places 2 or fewer fractions in the correct columns.</td>
</tr>
</tbody>
</table>
1. Place these fractions in order from least to the greatest on the number line below

\[
\frac{5}{3} \quad \frac{9}{4} \quad \frac{6}{6} \quad \frac{5}{12} \quad \frac{1}{2}
\]

2. Yamen and his pal Hun Sung decided they needed to get into shape. Yamen ran \(\frac{5}{10}\) of a mile while Hun Sung ran \(\frac{4}{5}\) of a mile.

   a. Determine who ran the furthest distance and explain your answer.

   b. Estimate the total distance both students ran.

3. Place the following fractions in the appropriate columns:

   \[
   \frac{3}{8} \quad \frac{5}{6} \quad \frac{5}{4} \quad \frac{5}{3} \quad \frac{1}{8}
   \]

   Closer to 0  Closer to \(\frac{1}{2}\)  Closer to 1  Closer to 2
4. Compare \( \frac{7}{4} \) to \( \frac{3}{2} \). Use words and pictures to explain which is larger.

5. Compare using <, > or =

   a. \( \frac{4}{10} \) \( \frac{9}{16} \)  
   b. \( \frac{1}{2} \) \( \frac{12}{24} \)  
   c. \( \frac{3}{4} \) \( \frac{5}{6} \)  
   d. \( \frac{3}{2} \) \( \frac{4}{3} \)

6. Hannah needed seven-eighths of a yard of string to make a belt and three-fourths of a yard of string to make a necklace. About how much string does Hannah need to make a belt and a necklace?

7. Jay used \( \frac{2}{3} \) cup of chocolate chips in his pancake recipe and \( 1 \frac{3}{4} \) cup of chocolate chips in his brownie recipe. Estimate the amount of chocolate chips Jay used for both recipes?
### Question 1 Performance Level Descriptors

<table>
<thead>
<tr>
<th>Points</th>
<th>Performance Level Descriptors</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>2 points</strong></td>
<td><strong>Adequate Understanding</strong></td>
</tr>
<tr>
<td></td>
<td>Places all five fractions in order on the number line</td>
</tr>
<tr>
<td></td>
<td>For example:</td>
</tr>
</tbody>
</table>
|              | \[
|              | \frac{5}{12}, \frac{1}{2}, \frac{6}{6}, \frac{5}{3}, \frac{9}{4}.                                                                                           |
| **1 point**  | **Partial Understanding**                                                                                                                                          |
|              | Places 3 of the 5 fractions in order on the number line.                                                                                                          |
|              | For example: Places fractions ≤ 1: \[
|              | \frac{5}{12}, \frac{1}{2}, \frac{6}{6}.                                                                                                                       |
| **0 points** | **Limited Understanding**                                                                                                                                          |
|              | Places 2 or fewer fractions in order on the number line.                                                                                                       |
|              | *Focus intervention on estimating fractions to halves or whole numbers using number lines, physical models and visual representations.*                        |

### Question 2 Performance Level Descriptors

<table>
<thead>
<tr>
<th>Points</th>
<th>Performance Level Descriptors</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>2 points</strong></td>
<td>Identifies Hun Sung as running the farthest and gives an appropriate explanation. Estimates the distance at a mile and a half.</td>
</tr>
</tbody>
</table>
|              | For example: \[
|              | \frac{4}{5} \text{ is greater than } \frac{4}{10}. I know this because 5 is half of 10. 4 is more than half of 5, so \frac{4}{5} \text{ is greater than } \frac{4}{10}. |
| **1 point**  | Identifies Hun Sung as running the farthest but does not give an explanation.                                                                                |
| **0 points** | Identifies Yamen as running the farthest.                                                                                                                     |

### Question 3 Performance Level Descriptors

<table>
<thead>
<tr>
<th>Points</th>
<th>Performance Level Descriptors</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>2 points</strong></td>
<td>Places all 5 fractions in the correct column.</td>
</tr>
<tr>
<td></td>
<td>For example:</td>
</tr>
</tbody>
</table>
|              | \[
|              | \frac{1}{8}, \frac{3}{8}, \frac{5}{6}, \frac{5}{4}, \frac{5}{3}.                                                                                           |
| **1 point**  | Places 3 - 4 of the 5 fractions in the correct column.                                                                                                          |
| **0 points** | Places 2 or fewer fractions in the correct column.                                                                                                              |
Question 4
Concludes $\frac{2}{3}$ is equal to $1\frac{1}{3}$ while $\frac{4}{7}$ is equal to $1\frac{1}{7}$, so that means $1\frac{1}{3}$ is larger than $1\frac{1}{7}$. The student can also draw a visual model showing the two fractions and concludes that $\frac{2}{3}$ is larger than $\frac{4}{7}$.

Question 5
a. $\frac{4}{10} < \frac{9}{10}$
   b. $\frac{1}{2} = \frac{12}{24}$
   c. $\frac{3}{4} < \frac{5}{6}$
   d. $\frac{3}{2} > \frac{4}{3}$

Question 6
About 2 yards, but will be less.

Question 7
About $2\frac{1}{2}$ cups of chocolate chips.