Linear Functions: Are You Ready for the REAL World? – Grade Ten

Lesson Summary:
In small groups students emulate a real world problem and model it with mathematics. Observe and facilitate discussion as the groups use various strategies, such as (graphs, brainstorming, tables, and dialogue) Students identify variables, recognize the relationship of two variables and create the ordered pairs to derive a linear function. Students identify the slope and the intercepts of this function and interpret them according to the problem. Students graph the linear function, recognize the slope and the intercepts in the graph. The slope of the cost function is the unit cost of the items and the y-intercept is the overhead. The slope of the revenue function is the price per unit sold, the y-intercept is the beginning costs incurred to sell (license), and the x-intercept is the break-even point.

Estimated Duration: Three hours and 20 minutes

Commentary:
This lesson uses phases of the Five-E Model, a learning cycle which supports research in brain-based learning and standards based education. In initial phases students engage in the world of business in a motivating context to provide evidence of students’ prior knowledge. Students continue to explore mathematical concepts through activities and discussion. Using real-world contexts allows students to extend the concepts beyond the classroom. Continually evaluate and monitor student understanding in a standards-based environment.

Instructional Tip:
Students write an equation for two accounts in a business simulation. Before the lesson, students should be able to create and graph linear equations and to manipulate any linear equation into slope-intercept, point-slope and/or standard form. Students create graphs of each equation for the visualization of the function and explain the parts of the equation represented on the graph. Understanding how linear equations represent relations in the real world is a prerequisite to understanding higher-level equations and their applications.
Pre-Assessment:
Distribute worksheet Pre-Assessment, Attachment A, and allow the students to work on it individually for 20 minutes. Collect the work and review it for misconceptions and common errors. (Individual scores/grades are not necessary for this pre-assessment.) Note any exceptional work and ideas.

Scoring Guidelines:
• Informally observe the class and check the students’ work afterwards.
• Based on the review of student work, review mathematical concepts that address the common misconceptions or errors.
• Students able to answer all questions of the pre-assessment without error may proceed directly to the exercises and then to the lesson extensions.
• Students having extreme difficulty completing the pre-assessment may need additional intervention on equations.

Post-Assessment:
Distribute the Post-Assessment Worksheet, Attachment C, to students.
Create a mathematical model from the given situation. Print the scenario for each student or use an overhead projector.

A local firm wants to give incentives to entice hard working people to work at its business. They are offering a 25 cent raise for each pay period that the new employee stays with the business. If each pay period is two weeks and the starting pay is $4.65 an hour, how long will it take a new hire to reach $11.00 an hour? Create a mathematical model by defining the variables, writing a linear equation and creating a graph for the relation described. Explain what the x-intercept, y-intercept and the slope represent in this situation.

Scoring Guidelines:
Use 4-point rubric for scoring:
4 Develops an accurate equation and clearly describes the interpretation of the equation and the problem. Checks to see if the relation makes sense by verifying specific values for the variables.
  • For example, how much will the employee make at the end of two pay periods?
  • Describe the slope, the y-intercept and the x-intercept accurately according to the problem.

3 Develops an equation that does not quite fit the data. Shows an understanding of the process, but makes a minor error in one of the steps in the task resulting in an incorrect equation. However, describes interpretation of the slope and its intercepts correctly, based on the wrong equation. The graph is correct and consistent with the error.
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2 Makes a table of values fitting the problem, but is unable to create the equation. Attempts to describe the interpretations, but the meaning is unclear. The graph for the data is accurate.

1 Creates only the tables, unable to create the graph or equation. Cannot describe the interpretation of the equation or intercepts.

0 Makes no attempt to create a graph, equation, interpretation or table. Copies information from the problem and makes no attempt to use the information to solve the problem.

**Instructional Procedures:**

**Part One**

“Manufacture” candy for retail by buying bulk, repackaging and then selling the processed product.

1. Introduce the activity with comments and/or questions to get the students thinking of running a manufacturing business. (Engage)
   - What is the purpose of business?
   - What basic things must be in place to have a business?

**Instructional Tips:**
This is a good opportunity to talk about various reasons for owning a business. Is the only purpose of the business to make money or are there additional purposes? Is the motivation the satisfaction of producing a needed product, personal satisfaction or strictly monetary satisfaction? Introduce basic economic terms such as cost, revenue and profit. A discussion about the meaning of profit should lead to the definition of profit as revenue minus cost. Production is creating a product which the consumer wants. Discuss demand versus supply. How are prices related to demand and supply? What historical events occurred related to the problems/benefits of supply and demand? (Gasoline, children’s toys, etc.)

2. Formation of businesses (Engage, Explore)
   - Create small groups of three or four students. Pre-select these groups to create an optimum learning environment.
   - Identify roles for each member, such as accountant, recorder, worker, etc. More can be created as the need arises in the business.

**Instructional Tips:**
A. Each group identifies a recorder and discusses the roles of the other members in the business. Remind students the group is responsible for a good final product, regardless of the group’s choice. [Extension: Have students identify a business, and research the positions in that business, the job descriptions and the salary ranges for the positions. Comparisons could be made by what is required of one business, but not another.]
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B. Groups need to keep detailed records, identifying all transactions. As students identify the variables used, ensure that the number of items purchased, cost of items purchased, revenue and time involved are included. Distribute the Blank Lesson Worksheet, Attachment E, the tables used to identify costs for bulk items and suggested retail prices. Change the outcome (if desired) by giving the groups different amounts of money to start their business.

3. Manufacturing process. (Engage, Explore)
   Brainstorm to decide on packaging and pricing.
   • The business (students) buy from the wholesaler (teacher) large quantities of the raw material (large bags of candy).
   • The business processes the bulk raw material by packaging it into smaller retail packages.
   • The business sells the product to the public (the teacher).

Instructional Tip:
A. Have the large bags of candy ready to sell to students. Set a price for the bulk bag of candy. Student use the price per piece, not the gross price to determine the retail price. Define overhead cost as the initial money or cost of doing business, a one-time fee. Buy back the finished product at a set price after the students have processed it. (This price may vary with supply and demand.)
B. Set the prices and values for any or all items such as bulk price and retail price for each unit. However, allow students to set some prices so they can develop an understanding of costs versus revenues. Set up a competition to see which group earns the highest profit per member, but do not let it become the focus of the lesson. (If interested, discuss price gouging.)

4. Accounting records (Engage, Explore)
   Complete record sheet.
   • Raw materials purchased.
   • Cost per unit.
   • Costs of raw materials
   • Number of processed items sold.
   • Price per unit.
   • Revenue generated from the sale of the processed items.

5. Create Mathematical Models (Explore, Explain, Elaborate)
   Complete the Blank Lesson Worksheet, Attachment E. Use Sample Lesson Key, Attachment F, to check.
   • Cost function.
   • Revenue function.
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**Instructional Tip:**
Students make several tables of ordered pairs relating (1) the number of items purchased and their costs and (2) the number of items sold and the total revenue.

**Closure:**
To close the lesson, choose a method for display or reporting groups’ work.
- Each group reports to another group or to the entire class.
- Put results on the board and discuss these results with the class.
- Individually, students write what they have learned and questions that remain unanswered. (Read and respond to these.).

**Instructional Tip:**
Students report the slope of the cost function as the unit cost, and the $y$-intercept as the initial cost. The slope of the revenue function is the price per unit, the $y$-intercept is (negative) the cost of the vendor’s license and the $x$-intercept is the break-even point.

**Part Two (Elaborate)**


**Instructional Tip:**
Decide whether to raise issues relating the real-world context to the mathematical model now or after the graphing activity. This may be a good opportunity to discuss the concept of discrete versus continuous data. Only Problem 4 is a continuous function and the other problems are discrete. Discuss how using a mathematical model to represent a real-world situation may lead to some inappropriate solutions when the context is ignored.

7. In pairs, create mathematical models for each problem.
   - Graph and table
   - Equation
   - Interpretation of slope and intercepts

**Instructional Tips for exercise problems:**
Complete the activity as a homework assignment if more time is needed. Students who have difficulty with Part One may work only the beginning problems using the worksheet similar to the *Post-Assessment Worksheet*, Attachment C. Students who master Part One can move quickly to the latter problems and/or teach or assist those having difficulty. (The last problem has several parts of linear equations connected together and is included as a challenge.)

A. Each problem gets progressively more involved in:
   - Length and information to read
   - Number of mathematical concepts involved
   - Complexity of the processes.
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B. Encourage students to work all the problems, but many students will need help with the latter problems. These samples provide a great opportunity for students to work together collaboratively to share their strengths.

C. Materials needed: graph paper, paper, pencils, and calculators if possible for the harder calculations, although only the four basic arithmetic operations are needed for this lesson.

**Instructional Tip:**
Use graphing calculators or other graphing software to graph the equations, or use computers to create spreadsheets for the tables.

**Vocabulary:**
- bulk
- costs
- equation
- graph
- intercepts
- manufacture
- overhead
- problem
- rates
- revenues
- slope
- spreadsheet
- table
- unit cost
- unit price

**Differentiated Instructional Support:**

**Standard:**
Define level of content understanding acceptable for various students. (know, understand, apply, analyze, evaluate, create)

- Realize students who have difficulty with these skills and concepts have difficulty creating tables, graphing points and calculating slope from the definition.
- Students who have basic understanding can analyze the graph, table or problem for interpretations of the slope, and the intercepts.
- Students who have a strong understanding can use linear equations to model real world situations.

**Instruction:**
Identify aspects of instructional strategies that align to student interest, learning style and level of understanding

- Students relate easily to money.
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- Interactive or hands-on activities appeal to more learning modes.
- Four ways to model a real world problem allow for differentiated learning styles.
  - Graph – visual-spatial
  - Table – naturalist
  - Equation – mathematical-logical
  - Problem – verbal-linguistics
- Simulation of a business
  - Working in a group - interpersonal and verbal-linguistics
  - Manufacturing – bodily-kinesthetic

Assessment:
- Students have several opportunities to both develop and describe in the pre-assessment and the post-assessment.
- Opportunities exist to respond using different learning styles via the use of tables, graphs and equations.

Additional Support:
- The Exercise Problems, Attachment G, allow for increased difficulty and further research into multiple linear equations and introduction into linear equations and inequalities.
- Describe what would change in the cost function if the local government subsidized a business with monetary grants.
- Describe what would happen to your profit if the demand for the product went down. Speculate how the business could respond.
- Describe the graph if a graduated pricing plan is offered to the retailers. For example, if someone ordered at least 50 items, the item price is reduced. Describe a situation where the dependent value would decrease as the independent value increased. (The graph decreases as it goes to the right.)
- Graph the cost equation and the revenue equation on the same grid, and describe the intersection of the two lines.

Extensions:
- Research business operations and report on responsibilities of different departments.
- Explore interest problems by graphing savings or loan account balances over time.
- Explore economic conditions in the newspaper or history book and report on rates, graphs, tables and equations found.
- Check the Internet for further exploration with linear equations, graphs and intercepts. Report on other uses of slope and intercepts.
- In what kinds of situations would the graph be a curve? Describe the equation.

Home Connections:
Discuss business situations with other adults to find linear relations in everyday situations.

Interdisciplinary Connections:
Basic business principles and economics emerge throughout the lesson.
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Social Studies: Economics
Students use economic reasoning skills and knowledge of major economic concepts, issues and systems in order to make informed choices as producers, consumers, savers, investors, workers and citizens in an interdependent world.

**Benchmark A:**
Compare how different economic systems answer the fundamental economic questions of what goods and services to produce, how to produce them, and who will consume them.

**Benchmark B:**
Explain how the U.S. government provides public services, redistributes income, regulates economic activity, and promotes economic growth and stability.

**Materials and Resources:**
The inclusion of a specific resource in any lesson formulated by the Ohio Department of Education should not be interpreted as an endorsement of that particular resource, or any of its contents, by the Ohio Department of Education. The Ohio Department of Education does not endorse any particular resource. The Web addresses listed are for a given site’s main page, therefore, it may be necessary to search within that site to find the specific information required for a given lesson. Please note that information published on the Internet changes over time, therefore the links provided may no longer contain the specific information related to a given lesson. Teachers are advised to preview all sites before using them with students.

*For the teacher:* Large bags with many small pieces of colored candies (beans or another large quantity of small objects), sandwich bags, play money (if not available keep a register of expenses), graph paper, worksheets

*For the student:* Pencil and paper, calculator (if necessary)

**Research Connections:**

**Attachments:**
Attachment A, *Pre-Assessment*
Attachment B, *Pre-Assessment Key*
Attachment C, *Post-Assessment Worksheet*
Attachment D, *Post-Assessment Worksheet Key*
Attachment E, *Blank Lesson Worksheet*
Attachment F, *Sample Lesson Key*
Attachment G, *Exercise Problems*
Attachment H, *Exercise Problem Key*
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Attachment A
Pre-Assessment

Linear equations.

1. Graph the line containing the following points:

<table>
<thead>
<tr>
<th>X</th>
<th>2</th>
<th>-5</th>
<th>8</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y</td>
<td>7</td>
<td>-7</td>
<td>19</td>
<td>23</td>
</tr>
</tbody>
</table>

2. Graph \( y = -\frac{1}{3}x + 4 \)

3. Write the equation of the line through the points (1, 2) and (-2, 3) in slope-intercept form.

4. For \( 2x + 3y = 12 \),
   a. Coordinates of the x-intercept
   b. Coordinates of the y-intercept
   c. Value of the slope

5. A salesman starts a trip with 15 gallons of gasoline in his tank. After traveling 200 miles he has 10 gallons of gasoline in his tank. Model this relationship with a linear equation relating the number of miles you can travel to the remaining gallons of gasoline. Explain what the x-intercept, the y-intercept, and the slope represent in this situation.
1. \( y = 2x + 3 \)

2. \( y = -\frac{1}{3}x + 4 \)
3. \( y = \frac{5}{3} x + \frac{1}{3} \)

4. \( y = -\frac{2}{3} x + 4 \)
   a. (6,0)
   b. (0,4)
   c. \( \frac{2}{3} \)

5. \( x = \) Gallons of gas
   \( y = \) Miles traveled

   \( y = -40x + 600 \)

   The slope -40 is the miles per gallon, negative because the gas is being consumed.
   The 600 \( y \)-intercept is the number of miles you can travel before the gas is gone
   The \( x \)-intercept is the capacity of the tank when the first measurement is made.
Attachment C - Post-Assessment Worksheet

Set Values

Define Variables:

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<td></td>
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</tr>
</tbody>
</table>

Tables:

<p>| | |</p>
<table>
<thead>
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<th></th>
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</thead>
<tbody>
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<td></td>
<td></td>
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<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Graph each table on graph paper.
Label graphs appropriately.

Calculate:

Name two points
- 
- 

Slope

Equation
- 
-

Describe the slope and the y-intercept for each equation as it relates the situation.
Let \( x = \) number of pay periods
\( y = \) pay rate

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$4.65</td>
</tr>
<tr>
<td>1</td>
<td>$4.90</td>
</tr>
<tr>
<td>2</td>
<td>$5.15</td>
</tr>
<tr>
<td>3</td>
<td>$5.40</td>
</tr>
<tr>
<td>4</td>
<td>$5.65</td>
</tr>
<tr>
<td>5</td>
<td>$5.90</td>
</tr>
<tr>
<td>6</td>
<td>$6.15</td>
</tr>
<tr>
<td>7</td>
<td>$6.40</td>
</tr>
<tr>
<td>8</td>
<td>$6.65</td>
</tr>
<tr>
<td>9</td>
<td>$6.90</td>
</tr>
</tbody>
</table>

Calculate slope using two points, \((0, 4.65)\) and \((1, 4.90)\)
Slope \( m = \frac{4.90 - 4.65}{1 - 0} = 0.25 \)

(Student may know that the pay rate is the same as the slope.)

Write an equation
Standard form \(25x - 100y = -465\)
Point-slope form \(y - 4.90 = 0.25(x - 1)\)
Solved for \( y \) \(y = 0.25x - 0.25 + 4.90\)
Slope-intercept form \(y = 0.25x + 4.65\)

Interpret the equation
The slope is the rate of pay increase, $0.25 per pay period
The \( y \)-intercept is the starting pay, $4.65 per hour
The \( x \)-intercept is not relevant to this situation.

Answer the question
To find how many pay periods it will take to reach $11 per hour
Let \( y = 11.00 \) and solve for \( x \)
\[ 11 = 0.25x + 4.65 \]
\[ 6.35 = 0.25x \]
\[ 25.4 = x \]
This means at the end of the 26th pay period the employee would earn more than $11.00 per hour. Because each pay period is 2 weeks, it would take 2 times 26, or 52 weeks before the employee’s hourly rate would go over $11 per hour.
Graph the function

\[ y = 0.25x + 4.65 \]
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Attachment E
Blank Lesson Worksheet

Cost Function
Set Values

__________ = Overhead cost
__________ = Cost of bulk for _______ pieces

Define Variables:

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>________</td>
<td>Number of items</td>
</tr>
<tr>
<td>________</td>
<td>Total cost of items purchased incl. overhead</td>
</tr>
</tbody>
</table>

Tables:

<table>
<thead>
<tr>
<th>Purchase</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Number of Items Purchased</th>
<th>Total Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Calculate:

Cost

<table>
<thead>
<tr>
<th>Name two points</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

Slope

Equation

Describe the slope and the y-intercept for the situation.
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Attachment E (Continued)
Blank Lesson Worksheet

**Revenue Function**

**Set Values**

\[ \text{ } = \text{ Price per package sold} \\
\text{ } = \text{ Vendor’s License} \\

**Define Variables:**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>\text{ }</td>
<td>Number of items produced</td>
</tr>
<tr>
<td>\text{ }</td>
<td>Total Revenue</td>
</tr>
</tbody>
</table>

**Tables:**

**Purchase**

<table>
<thead>
<tr>
<th>Number of Items Purchased</th>
<th>Total Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
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<td></td>
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<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Graph each table on graph paper  
Label your graphs  
Label graphs appropriately

**Calculate:**

<table>
<thead>
<tr>
<th>Name two points</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Slope</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
</tbody>
</table>

Describe the slope and the y-intercept for the situation.
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Attachment F
Sample Lesson Key

Cost Function
Set Values

<table>
<thead>
<tr>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10.00</td>
<td>Overhead Cost</td>
</tr>
<tr>
<td>$  8.00</td>
<td>Cost of bulk (for 50 pieces)</td>
</tr>
</tbody>
</table>

Define variables:

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>Number of items purchased</td>
</tr>
<tr>
<td>C(x)</td>
<td>Total Cost of items purchased</td>
</tr>
</tbody>
</table>

Total

Tables: Cost

<table>
<thead>
<tr>
<th>Number of items purchased</th>
<th>Total Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>C(x)</td>
</tr>
<tr>
<td>0</td>
<td>$10.00</td>
</tr>
<tr>
<td>1</td>
<td>$10.16</td>
</tr>
<tr>
<td>10</td>
<td>$11.60</td>
</tr>
<tr>
<td>50</td>
<td>$18.00</td>
</tr>
<tr>
<td>100</td>
<td>$26.00</td>
</tr>
<tr>
<td>200</td>
<td>$42.00</td>
</tr>
</tbody>
</table>

Graph each table on graph paper. Label graphs appropriately.

Cost Function

![Graph of Cost Function](image)
Calculate:

<table>
<thead>
<tr>
<th>Name two points</th>
<th>(0, 10) and (50, 18)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slope</td>
<td>( m = \frac{(18 - 10)}{(50 - 0)} )</td>
</tr>
<tr>
<td></td>
<td>( = \frac{4}{25} = .16 )</td>
</tr>
<tr>
<td>Equation</td>
<td>( y = .16x + 10 )</td>
</tr>
</tbody>
</table>

Explain what the slope and the \( y \)-intercept means for each of the graphs. Cost: Slope is the cost per item and the \( y \)-intercept is the overhead.
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Attachment F (Continued)

Recovery Function

Set Values

$0.45 = \text{Price per package sold}$

$5.00 = \text{Vendor’s license needed to sell the product in town.}$

Define variables:

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n$</td>
<td>Number of items produced</td>
</tr>
<tr>
<td>$R(n)$</td>
<td>Total Revenue</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Number of items produced</th>
<th>Total Revenue</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n$</td>
<td>$R(n)$</td>
</tr>
<tr>
<td>0</td>
<td>-$5.00</td>
</tr>
<tr>
<td>10</td>
<td>-$0.50</td>
</tr>
<tr>
<td>20</td>
<td>$4.00</td>
</tr>
<tr>
<td>30</td>
<td>$8.50</td>
</tr>
<tr>
<td>40</td>
<td>$13.00</td>
</tr>
<tr>
<td>50</td>
<td>$17.50</td>
</tr>
</tbody>
</table>

Calculate:

Name two points
(0,-5) and (20,4)

Slope

$$m = \frac{4 - (-5)}{20 - 0} = \frac{9}{20} = 0.45$$

Equation

$$y = 0.45x - 5$$

Explain what the slope and the $y$-intercept means. Slope is the price per item sold and $y$-intercept is the cost of the vendor’s license to sell. Revenue: the product
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Attachment G
Exercise Problems

Make a model for each situation. Generate an equation for each problem and interpret the slope and the intercepts of the graph of the equation.

1. Money collected from the sales of lemonade for 50 cents a glass.

2. A local car dealer sells cars for $10,000 each. He must pay a lease on his place of business at the rate of $1000 each day. Make a model to represent his daily revenue based on the number of cars sold, if there are no other expenses.

3. A local restaurant pays a monthly lease of $3500. If the restaurant averages 125 customers daily and makes $4.25 on each customer, how many customers must the restaurant serve before they make a profit?

4. A local township trustee is responsible for displaying high water signs when a local creek floods the road during a steady rain. Being an astute mathematics student she realizes that she can predict with fair accuracy the time the water will overflow its banks and reach across the road. But because she likes her sleep, the trustee gathers data and goes to bed knowing she will set her alarm and get up one hour before the road floods and set up her warning signs for constituents. At 8:00 p.m. she read the water level at 22.1 feet. Later at 10:30 p.m. she reads the water level at 23.7 feet. With the weather forecast predicting consistent rain patterns for the area and knowing that the road will flood at 27.4 feet, for what time must she set her alarm?

5. A local town council provided $10,000 to a promoter to bring a popular performing group to town. The promoter receives $30 a ticket for the best seats, 1 through 100; $20 a ticket for the economical seats, 101 through 300; and $10 a ticket for lawn seating, 301 through 500. How much revenue will the promoter receive, if all the tickets were sold? What was the average revenue for each ticket sold?
Exercise Problems Key

1. Let $n$ = the number of glasses sold
   $R(n)$ = the revenues generated
   $R(n) = 0.50n$

   The slope is the cost per glass. The $y$-intercept is zero and indicates no money is made until a glass is sold. The $x$-intercept is zero and means that zero money is made from zero glasses sold.

2. Let $n$ = number of cars sold
   $R(n)$ = the revenues from the cars sold
   $R(n) = 10000n - 1000$

   The slope represents the price per car, and the $y$-intercept is the overhead (the lease). The $x$-intercept is the number of cars that must be sold to break even, in this case $\frac{1}{10}$ of a car.
3. Let $x$ = the number of customers served
   $P(x)$ = the total profit made during the month
   $R(x) = 4.25x - 3500$
   The slope is the profit made per customer and the 3500 is the monthly lease payment, normally referred to as the overhead. The $x$-intercept is the break even point and occurs on the $824^{\text{th}}$ customer. This would happen on the $7^{\text{th}}$ day.

4. Let $t$ = the number of hours from 8:00 p.m.
   $F(t)$ = the height of the water in feet
   $F(t) = .64t + 22.1$
   The slope is the rate that the water is rising. The $y$-intercept is the beginning water height. The road will flood with $F(t)$ 27.4 feet, which is the same as when $t = 8.28$ hours. Therefore, the township trustee must set her alarm for one hour before, or 7.28 hours after 8:00 p.m. She must set her alarm for about 3:17 a.m.
5. Let $n$ = the number of the tickets sold.
Total revenue is:
$R(n) = 30n + 10,000$ if $0 \leq n \leq 100$
$R(n) = 20(n-100) + 13,000$ if $101 \leq n \leq 300$
$R(n) = 10(n-300) + 17,000$ if $301 \leq n \leq 500$

The slope in each equation is the revenue per ticket. The $y$-intercept of $10,000$ is the beginning revenue. The $x$-intercept is meaningless to this problem since it is negative. The total revenue generated is $19,000. The average revenue received for each ticket is $38.