Exploring Function Graphs – Grade 10

### Ohio Standards Connection

**Patterns, Functions, and Algebra**

**Benchmark B**
Identify and classify functions as linear or nonlinear, and contrast their properties using tables, graphs or equations.

**Indicator 1**
Define function formally and with f(x) notation.

**Indicator 2**
Describe and compare characteristics of the following families of functions: square root, cubic, absolute value and basic trigonometric functions; e.g., general shape, possible number of roots, domain and range.

**Benchmark A**
Generalize and explain patterns and sequences in order to find the next term and the nth term.

**Mathematical Processes Benchmark**
E. Use a variety of mathematical representations flexibly and appropriately to organize, record and communicate mathematical ideas.

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**Lesson Summary:**
Students investigate three different sets of data that relate temperature to another variable in the physical world. These illustrate linear, inverse and exponential relationships. In each set of data students look for patterns and predict what will occur at a given higher or lower temperature. They graph the data set, describe the characteristics of the graph, and identify what type of function the graph represents. At that point they may alter their predictions based upon their graphs. After exploring all three sets of data, the students look for similarities and differences in the data and graphs.

**Estimated Duration:** Two hours

**Commentary:**
This lesson provides opportunities for students to discuss temperature changes in non-routine situations while looking at the characteristics of the graphs of these situations. Students should be able to graph linear functions, exponential functions and inverse variation from given data. By recognizing the type of function, students make predictions based on those functions and/or graphs. Integrate technology in this lesson as possible.

**Pre-Assessment:**
- Students work in groups to complete Pre-Assessment Worksheet, Attachment A, which asks them to match parent functions and tables with the graphs. Students self-check for accuracy with answers posted on the board or overhead projector.
- Each group takes one of the functions and describes the unique characteristics of the curve for that function.
- Each group posts the function, graph and characteristics of the curve on chart paper and reports its findings to the whole class.
- With all charts posted, the class discusses what unique feature of a function causes a certain behavior of the curve. Together the class determines which graphs/equations are linear, indirect and exponential.
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Instructional Tip:
Point out the difference between a linear relationship with negative slope and a relationship of indirect variation, as necessary.

Scoring Guidelines:
This pre-assessment does not need to be scored. Review parent functions and determine if students have a good understanding of the graphs. Check the matching worksheet, monitor for accuracy in the group reports, and conduct a class discussion to determine if intervention is necessary before proceeding with the lesson.

Post-Assessment:
Students work individually to investigate two sets of real data, predict new data points and determine what type of function each represents.

- Distribute Post-Assessment Worksheet, Attachment B, to students. This contains two sets of data: (1) the number of cricket chirps vs. temperature and (2) cold drink temperature vs. time. Ask students to make predictions for points not in the table.
- Students look at the data, generate a graph of the data and determine what type of function the data represents.
- Students justify their decisions with written explanations.

Scoring Guidelines:
Rubric for each set of data:
4 The student correctly
- predicts new data points;
- identifies the function represented; and
- demonstrates an understanding of the function in written explanation.
3 The student completes 2 of the 3 objectives above.
2 The student completes 1 of the 3 objectives above.
1 The student shows some understanding of the function.

Instructional Procedures:
1. Give students the three data sets, Lesson Worksheet, Attachment C. Have students work in groups to investigate three different sets of data: (1) water temperature vs. ocean depth, (2) coffee temperature vs. time, and (3) temperature vs. elevation on a mountain. Ask them to observe and discuss each data set and its characteristics, as well as generate a graph of the data set. Graph paper or graphing calculators may be used, depending on student’s background and availability of technology. These are examples of direct variation (linear function), inverse variation and an exponential function.

Instructional Tip:
It may be necessary to discuss how to identify independent and dependent variables before the graphing is done.

2. Have students complete Part One for each of the three data sets.
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Instructional Tips:
- Students with complete understanding will organize and present their graphs and answers by their own method of choice.
- Optional worksheets, *Lesson Problem #1*, Attachment D; *Lesson Problem #2*, Attachment E and *Lesson Problem #3*, Attachment F, are for those students who need help in organizing their thoughts or for each group to use for reporting.

3. Have students write a reflection demonstrating what they have learned about direct, inverse and exponential relationships. Be sure they include similarities and differences between the different functions and their graphs.

Differentiated Instructional Support:
Instruction is differentiated according to learner needs, to help all learners either meet the intent of the specified indicator(s) or, if the indicator is already met, to advance beyond the specified indicator(s).
- Organize groups to allow all students to contribute according to their strengths and learn from others according to their needs.
- All students should be able to name the function and talk about the characteristics of the graph. Expect varying levels of reasoning in connecting the graph to the nuances of the data, connecting the graph to the problem situation and in comparing the functions to each other.
- Differentiation can occur in the homework. (See Home Connection)

Extensions:
- The same procedures in this lesson can be used in the future study of new functions, i.e., cubic, quadratic, square root.
- Students can predict an equation that would model the data given for each function and test their prediction.

Home Connection:
- If some students have difficulty interpreting tables and graphing, additional practice could be given as homework.
- For students that have already mastered recognizing the three types of functions from equations, tables and graphs, their homework could be to find additional data sets representing relationships.

Interdisciplinary Connections:
Science
Standard: Scientific Inquiry
Benchmark(s):
A. Participate in and apply the processes of scientific investigation to create models and to design, conduct, evaluate, and communicate the results of these investigations.
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Indicator(s):
3. Use mathematical models to predict and analyze natural phenomena.
4. Draw conclusions from inquiries based on scientific knowledge and principles, the use of logic and evidence (data) from investigations.

Materials and Resources:
The inclusion of a specific resource in any lesson formulated by the Ohio Department of Education should not be interpreted as an endorsement of that particular resource, or any of its contents, by the Ohio Department of Education. The Ohio Department of Education does not endorse any particular resource. The Web addresses listed are for a given site’s main page, therefore, it may be necessary to search within that site to find the specific information required for a given lesson. Please note that information published on the Internet changes over time, therefore the links provided may no longer contain the specific information related to a given lesson. Teachers are advised to preview all sites before using them with students.

For the teacher:  Worksheets, teacher notes

For the student:  Worksheets, graph paper

Vocabulary:
• dependent variable
• direct variation
• domain
• exponential decay
• exponential growth
• function
• independent variable
• inverse variation
• range

Technology Connections:
Students plot data using a graphing calculator, determine what function the data represents and find the regression equation for that data set. The equation can then be used to predict a new data point.

Research Connections:

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Attachments:
Attachment A, Pre-Assessment Worksheet with Answer Key
Attachment B, Post-Assessment Worksheet with Answer Key
Attachment C, Lesson Worksheet with Answer Key
Attachment D, Lesson Problem #1 with Answer Key
Attachment E, Lesson Problem #2 with Answer Key
Attachment F, Lesson Problem #3 with Answer Key
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Attachment A
Pre-Assessment Worksheet
Directions: Match the function with its table and graph.

1. \( y = 2x \)

   A. \[
   \begin{array}{c|c}
   x & y \\
   \hline
   -2 & 0.25 \\
   0 & 1 \\
   2 & 4 \\
   4 & 16 \\
   6 & 64 \\
   \end{array}
   \]

2. \( f(x) = 2^x \)

   B. \[
   \begin{array}{c|c}
   x & y \\
   \hline
   -2 & 1 \\
   0 & \text{No sol.} \\
   2 & -1 \\
   4 & -0.5 \\
   6 & -0.333 \\
   \end{array}
   \]

3. \( y = -2x \)

   C. \[
   \begin{array}{c|c}
   x & y \\
   \hline
   -2 & -4 \\
   0 & 0 \\
   2 & 4 \\
   4 & 8 \\
   6 & 12 \\
   \end{array}
   \]

4. \( f(x) = \frac{2}{x} \)

   D. \[
   \begin{array}{c|c}
   x & y \\
   \hline
   -4 & 16 \\
   -2 & 4 \\
   0 & 1 \\
   2 & 0.25 \\
   4 & 0.063 \\
   \end{array}
   \]

5. \( y = 2^{-x} \)

   E. \[
   \begin{array}{c|c}
   x & y \\
   \hline
   -2 & 4 \\
   0 & 0 \\
   2 & -4 \\
   4 & -8 \\
   6 & -12 \\
   \end{array}
   \]

6. \( y = -\frac{2}{x} \)

   F. \[
   \begin{array}{c|c}
   x & y \\
   \hline
   -2 & -1 \\
   0 & \text{No sol.} \\
   2 & 1 \\
   4 & 0.5 \\
   6 & 0.333 \\
   \end{array}
   \]
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**Attachment A (continued)**

**Pre-Assessment Worksheet**

**Answers:**

<table>
<thead>
<tr>
<th>Worksheet</th>
<th>1. &amp; 3. Linear</th>
<th>(form $y = ax$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2, A, II</td>
<td>2. &amp; 5. Exponential</td>
<td>(form $y = a^x$)</td>
</tr>
<tr>
<td>3, E, III</td>
<td>4. &amp; 6. Inverse</td>
<td>(form $y = \frac{a}{x}$)</td>
</tr>
<tr>
<td>4, F, V</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5, D, IV</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6, B, I</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Possible Answers**

**Function Descriptions**

**Exponential Functions**
- Is a single curve
- Rises or falls dramatically
- Rises when exponent is positive, falls when exponent is negative
- Never touches the $x$-axis
- Never goes below $x$-axis ($y$ is always positive)
- Does have a $y$-intercept (will be 1 unless there is a coefficient)
- Can represent growth or decay

**Linear Functions**
- Is a straight line
- Rises or falls at a constant rate (slope)
- Rises if coefficient is positive, falls if coefficient is negative
- Crosses the $x$- and/or $y$-axis

**Inverse Variation**
- Is made up of two curves that are symmetric
- Never crosses the $x$ or $y$ axis (no intercepts)
- Is positive in Quadrant I if the constant (numerator) is positive, and positive in Quadrant II if the constant is negative
- Gets very close to zero when $x$ is very large positive or very large negative.
I. The table shows the number of cricket chirps in a 15 second interval at various temperatures.

<table>
<thead>
<tr>
<th>Temp (°F)</th>
<th>Chirps per 15 sec</th>
</tr>
</thead>
<tbody>
<tr>
<td>54</td>
<td>15</td>
</tr>
<tr>
<td>65</td>
<td>21</td>
</tr>
<tr>
<td>68</td>
<td>23</td>
</tr>
<tr>
<td>79</td>
<td>31</td>
</tr>
<tr>
<td>82</td>
<td>33</td>
</tr>
<tr>
<td>89</td>
<td>38</td>
</tr>
</tbody>
</table>

1. Construct a graph of the data.

2. Determine what type of function the data represents. Explain how you arrived at your choice.

3. Estimate the number of chirps in a 15 second interval at 50°F.

4. Estimate the temperature when there are 17 chirps in a 15 second interval.
II. The table shows the temperature of a cold drink sitting out on the counter after the given number of minutes.

<table>
<thead>
<tr>
<th>Time (min)</th>
<th>Temp (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>11</td>
</tr>
<tr>
<td>43</td>
<td>15</td>
</tr>
<tr>
<td>87</td>
<td>18</td>
</tr>
<tr>
<td>117</td>
<td>19</td>
</tr>
<tr>
<td>160</td>
<td>20</td>
</tr>
<tr>
<td>180</td>
<td>20</td>
</tr>
</tbody>
</table>

1. Construct a graph of the data.

2. Determine what type of function the data represents. Explain how you arrived at your choice.

3. Estimate the temperature of the drink after 20 minutes.

4. Estimate the temperature of the drink after 3 hours.
Answer Key:

I. Cricket chirps
1. Graph of data table.

2. The function is linear (direct variation). The graph of data points appears to be linear, and the ratio of the change in temperature to the number of chirps is constant.
3. The number of chirps at a temperature of 50°F is approximately 12 chirps per 15-second interval.
4. The temperature at 17 chirps per 15-second interval is approximately 58°F.

II. Temperature
1. Graph of data table.

2. The function is exponential. The graph rises at first and then levels off to a constant (room temperature).
3. The temperature after 20 minutes would be approximately 13°C.
4. After 3 hours, the temperature would remain at 21°C.
1. The table shows the water temperature at various depths for part of the Pacific Ocean.

<table>
<thead>
<tr>
<th>Depth (m)</th>
<th>Temp (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>750</td>
<td>5</td>
</tr>
<tr>
<td>1000</td>
<td>4</td>
</tr>
<tr>
<td>2500</td>
<td>1.7</td>
</tr>
<tr>
<td>3700</td>
<td>1.2</td>
</tr>
<tr>
<td>4500</td>
<td>1</td>
</tr>
</tbody>
</table>

2. The table shows the temperature at different elevations on Mt. Everest in June.

<table>
<thead>
<tr>
<th>Height (ft)</th>
<th>Temp (°F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10,000</td>
<td>61</td>
</tr>
<tr>
<td>18,000</td>
<td>25</td>
</tr>
<tr>
<td>24,000</td>
<td>1</td>
</tr>
<tr>
<td>30,000</td>
<td>-27</td>
</tr>
</tbody>
</table>

3. The table shows the temperature of a cup of coffee cooling over time.

<table>
<thead>
<tr>
<th>Time (min.)</th>
<th>Temp (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>68</td>
</tr>
<tr>
<td>4</td>
<td>59</td>
</tr>
<tr>
<td>8</td>
<td>54</td>
</tr>
<tr>
<td>12</td>
<td>50</td>
</tr>
<tr>
<td>16</td>
<td>47</td>
</tr>
<tr>
<td>20</td>
<td>45</td>
</tr>
</tbody>
</table>
Part One
For each data set complete the following:

1. List characteristics of the data.

2. Identify the independent and dependent variables for the data set.

3. What type of relationship do you think the data represents and why?

4. Graph the data and describe the graph of the data set in your own words. (List as many characteristics as possible.)

5. What general type of function does the data set and its graph represent?

6. How does the graph describe the problem situation?

7. Give another example in real life that would have the same type of graph and explain why.

Part Two
Using your graph and knowledge of each function, predict the following:

Refer to problem #1
What will the temperature be at an ocean depth of 1500 m?

Refer to problem #2
At what height on Mt. Everest would the temperature be 40° F?
What would the temperature on Mr. Everest be at a height of 7000 ft?

Refer to problem #3
What would the temperature of the coffee be after 30 min?
After how many minutes would the coffee be 62° C?
Answer Key

Part One

#1 Data Set
1. As the depth increases, the temperature decreases, but not at a constant rate. The ratio of the change in temperature to change in depth decreases with depth.
2. The independent variable is ocean depth and the dependent variable is temperature.
3. Inverse relationship
4. A curve that is continually decreasing

5. Indirect variation. As one variable decreases, the other continues to decrease at a smaller rate.
6. It is getting colder as you go deeper in the ocean.
7. (Answers will vary.) As you age, you get shorter.

#2 Data Set
1. As the height increases, the temperature decreases at a fairly constant rate. The ratio of the change in temperature to change in height is constant.
2. The independent variable is height and the dependent variable is temperature.
3. Direct variation. As one variable increases the other decreases at a constant rate.
4. A straight line that is decreasing.

5. Linear function.
6. As you climb the mountain, it is getting colder.
7. (Answers will vary.) As you spend money, your bank account decreases.
1. As time increases, the temperature decreases at a smaller rate. The ratio of the change in temperature to change in time decreases with time.
2. The independent variable is time, and the dependent variable is temperature.
3. Exponential. As one variable increases, the other decreases dramatically.
4. A curve that decreases dramatically at first, and then levels off.

Exponential
5. As time goes on, the coffee cools and will eventually level off to room temperature.
6. (Answers will vary.) Over time the value of a car depreciates to $0.

Part Two
Refer to problem #1
   about 2.7°C

Refer to problem #2
   about 16,000 ft
   about 76°C

Refer to problem #3
   35°C
   2 min
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Attachment D
Lesson Problem #1

The table shows the water temperature at various depths for part of the Pacific Ocean.

<table>
<thead>
<tr>
<th>Depth (m)</th>
<th>Temp (°C)</th>
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<tbody>
<tr>
<td>750</td>
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</tr>
<tr>
<td>3700</td>
<td>1.2</td>
</tr>
<tr>
<td>4500</td>
<td>1</td>
</tr>
</tbody>
</table>

1. Graph the information in the table on graph paper.

2. Give two characteristics of the data.

3. Identify the independent and dependent variables.

4. What type of relationship do you think the data represents and why?

5. How does the graph describe the problem situation?

6. Using your graph and knowledge of each function, predict the following:
   What will the temperature be at an ocean depth of 1500m?
Answer Key

1. $Temperature \ (^{o}C)$

2. A curve that is continually decreasing. 
   As the depth increases, the temperature decreases, but not at a constant rate. 
   The ratio of the change in temperature to change in depth decreases with depth.

3. The independent variable is ocean depth and the dependent variable is temperature.

4. Inverse relationship

5. Indirect variation. As one variable decreases, the other continues to decrease at a smaller rate. 
   It is getting colder as you go deeper in the ocean.

6. about $2.7^{o}C$
The table shows the temperature at different elevations on Mt. Everest in June.

Height (ft) | Temp (°F)
---|---
10,000 | 61
18,000 | 25
24,000 | 1
30,000 | -27

1. Graph the information in the table on graph paper.

2. Give two characteristics of the data.

3. Identify the independent and dependent variables.

4. What type of relationship do you think the data represents and why?

5. How does the graph describe the problem situation?

6. Using your graph and knowledge of each function, predict the following:
   a. At what height on Mt. Everest would the temperature be 40°F?
   b. What would the temperature on Mt. Everest be at the height of 7000 ft?
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Attachment E (continued)
Lesson Problem #2

Answer Key

1. \( \text{Temperature (°C)} \)

2. Linear function
   A straight line that is decreasing
   As the height increases, the temperature decreases at a fairly constant rate.
   The ratio of the change in temperature to change in height is constant.

3. The independent variable is height and the independent variable is temperature.

4. Direct variation. As one variable increases the other decreases at a constant rate.

5. As you climb the mountain, it gets colder.

6. a. about 16000 ft
   b. about 76°F
The table shows the temperature of a cup of coffee cooling over time.

<table>
<thead>
<tr>
<th>Time (min.)</th>
<th>Temp (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
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</tr>
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<tr>
<td>12</td>
<td>50</td>
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<td>16</td>
<td>47</td>
</tr>
<tr>
<td>20</td>
<td>45</td>
</tr>
</tbody>
</table>

1. Graph the information in the table on graph paper.

2. Give two characteristics of the data.

3. Identify the independent and dependent variables.

4. What type of relationship do you think the data represents and why?

5. How does the graph describe the problem situation?

6. Using your graph and knowledge of each function, predict the following:
   a. What would the temperature of the coffee be after 30 min?
   b. After how many minutes would the coffee be 62°C?
Answer Key

1. Temperature (ºC)

2. A curve that decreases dramatically at first, and then levels off. As time increases, the temperature decreases at a smaller rate. The ratio of the change in temperature to change in time decreases with time.

3. The independent variable is time, and the dependent variable is temperature.

4. Exponential. As one variable increases, the other decreases dramatically.

5. As time goes on, the coffee is cooling and will eventually level off to room temperature.

6. a. 35ºC
   b. 2 min