Solving Problems Involving Chords, Radii, Tangents, Secants and Arcs within the Same Circle – Grade Ten

Lesson Summary:
In this lesson, students apply their knowledge of angles related to a circle, radii, secants, tangents and chords of a circle. They practice using formulas related to a circle, learned in previous lessons. They apply them to more realistic problem-solving situations. In some cases, students have to draw their own diagrams in order to solve the problem, which challenges students. Allow them to work in groups, and share ideas to provide them with different perspectives for solving the problems.

Estimated Duration: One hour and 20 minutes

Commentary:
Before beginning students must know the following: the definition of a circle, chords, tangents, secants, inscribed angles, central angles, and their respective properties as related to circles. In addition, the students need familiarity with the formulas that involve inscribed angles, tangent-chord angles, chord-chord angles, secant-secant angles, secant-tangent angles, tangent-tangent angles, angles intercepting the same or congruent arcs, angles inscribed in a semicircle and angles formed by intersecting chords. Questions in the Pre-Assessment Worksheet, Attachment A, are designed to identify the formulas that the students do not fully understand.

Pre-Assessment:
- Distribute Pre-Assessment Worksheet, Attachment A, to each student. This is intended to be an individual, non-graded activity to check the student’s background knowledge.
- Have students answer eight completion-type questions relating to arcs of a circle, using a paper-pencil method. Each problem requires calculations. Instruct students to show all related formulas and mathematical equations to justify answers.
- Select students to orally explain solutions to the class.

Ohio Standards Connection:

Geometry and Spatial Sense
Benchmark C
Recognize and apply angle relationships in situations involving intersecting lines, perpendicular lines and parallel lines.

Indicator 10
Solve problems involving chords, radii and arcs within the same circle.

Patterns, Functions, and Algebra
Benchmark F
Solve and graph linear equations and inequalities.

Indicator 10
Solve real-world problems that can be modeled using linear, quadratic, exponential or square root functions.

Mathematical Processes Benchmarks
A. Formulate a problem or mathematical model in response to a specific need or situation, determine information required to solve the problem, choose a method for obtaining this information, and set limits for acceptable solution.

Pre-Assessment Worksheet, Attachment A, is intended to be an individual, non-graded activity to check the student’s background knowledge.
Solving Problems Involving Chords, Radii, Tangents, Secants and Arcs within the Same Circle – Grade Ten

<table>
<thead>
<tr>
<th>Ohio Standards Connection:</th>
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<tbody>
<tr>
<td>B. Apply mathematical knowledge and skills routinely in other content areas and practical situations.</td>
</tr>
<tr>
<td>E. Use a variety of mathematical representations flexibly and appropriately to organize, record and communicate mathematical ideas.</td>
</tr>
<tr>
<td>F. Use precise mathematical language and notations to represent problem situations and mathematical ideas.</td>
</tr>
<tr>
<td>H. Locate and interpret mathematical information accurately, and communicate ideas, processes and solutions in a complete and easily understood manner.</td>
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</table>

**Scoring Guidelines:**
Calculations must be shown for the solution to be marked correct. An answer key is included in Attachment A.

**Number missed:**

<table>
<thead>
<tr>
<th>Number</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-2</td>
<td>Sufficient background knowledge; ready to begin lesson</td>
</tr>
<tr>
<td>3-4</td>
<td>Some review of background material required. The pre-assessment activity can be a tool to locate the problem areas, such as the correct use of the formulas, solving the equations correctly and finding the information requested.</td>
</tr>
<tr>
<td>5-8</td>
<td>Intervention is needed relating to the relationships of arcs, chords, tangents, and radii to a circle. Intervention may be needed for selecting and using the correct formulas and their subsequent solving.</td>
</tr>
</tbody>
</table>

**Post-Assessment:**
Solve questions relating to secants and tangents of a circle using paper and pencil. Calculations are required for both problems and a written explanation is required for the first problem. This is a self-check, non-graded assessment for the students to identify their weaknesses.

- Distribute the *Post-Assessment Worksheet*, Attachment B.
- Direct the students to solve the two problems and show all calculations.
- Tell the students to write a paragraph explaining the strategies they use to solve the first problem.
- Have students correct their own papers.

**Scoring Guidelines:**
An answer key is included with Attachment B. Score the two situations, awarding seven points for a perfect paper.

- For the first question, 1 point for correctly finding \( x \), 2 points for the calculations and 2 points for the written explanation.
- For question two, 1 point for the calculations and 1 point for finding the angle measure.
Solving Problems Involving Chords, Radii, Tangents, Secants and Arcs within the Same Circle – Grade Ten

Number missed:
0-2 Proficient
3-5 Some review of background material suggested
6-7 Intervention needed regarding the relationships of secants, tangents, and radii to a circle in a problem-solving situation.

Instructional Procedures:
1. Facilitate a class discussion, asking students to name several things they see in their daily lives which use circles in their design. Ask them to conjecture why they are circular.
2. Divide students into groups of three to four members and distribute Classroom Activity Problems, Attachment C.
3. Draw the graphic for the first problem on the board, overhead projector, digital projector or if prepared on a worksheet, distribute the worksheets to the student groups. Use graphing software, if available. Allow students sufficient time to complete the first problem. Make sure all groups have completed the first problem before continuing with the lesson.
4. Circulate around and observe students working. If they are experiencing difficulty and cannot continue, intervene and assist them by asking guiding questions.
5. Call on groups to represent and explain their solutions to the rest of the class. Allow the class to question the student after his/her explanation, and to offer different methods for solving.
6. Continue with the remaining problems, providing intervention to groups or individuals who misunderstand the skills and concepts.
7. Have the students choose a partner, using a “Partners-Meet” activity. Each is to tell his/her partner two new things he learned in class today that relate to circles, while the partner listens but does not comment. Then, they switch roles with their partner. Then, they write reflections in their notebooks about what they have learned.

Differentiated Instruction:
Differentiated instruction is addressed by the grouping of the students, allowing them to assist each other in solving the problems in the class activity. The teacher may choose to group them according to their understanding of the topic in order for students to work together and help each other.
- Use geometric software to help students visualize the angles or arcs in the problems.

Extension:
- Why are circles useful in industry in manufacturing items such as soup cans, bottles and pipes?
- Have the students find the total length of the belt, to the nearest inch, for homework problem 1, Homework Problems, Attachment D.
- Ask the students to determine why manhole covers are all circular.
Solving Problems Involving Chords, Radii, Tangents, Secants and Arcs within the Same Circle – Grade Ten

Home Connections:
- Assign problem-solving Homework Problems, Attachment D. An answer key is included with Attachment D.
- Identify how these concepts are used or seen within daily uses and in the environment.

Interdisciplinary Connections:
Science: How are circles important to machines, motors and wheels? What other fields rely on the use of a circular design? Why is the circle so influential?

Materials and Resources:
The inclusion of a specific resource in any lesson formulated by the Ohio Department of Education should not be interpreted as an endorsement of that particular resource, or any of its contents, by the Ohio Department of Education. The Ohio Department of Education does not endorse any particular resource. The Web addresses listed are for a given site’s main page, therefore, it may be necessary to search within that site to find the specific information required for a given lesson. Please note that information published on the Internet changes over time, therefore the links provided may no longer contain the specific information related to a given lesson. Teachers are advised to preview all sites before using them with students.

For the teacher: Overhead transparencies and overhead projector, computer with projection unit with graphing software may be used, or copies of the pre-assessment worksheet, classroom activity problems and post-assessment attachments for each student and group.

For the student: Pencil, paper; hand-held calculators may be used, but are not required.

Vocabulary:
- arc
- chord
- circle
- diameter
- equilateral triangle
- inscribed
- quadrilateral
- radius
- secant
- secant segment
- tangent
- tangent segment

Technology Connections:
Use calculators, computer with projection unit, overhead projector and calculators. If available, have students use dynamic geometry software.
Solving Problems Involving Chords, Radii, Tangents, Secants and Arcs within the Same Circle – Grade Ten

Researchers Connections:


General Tip:
For problems that do not have an associated diagram, have the students draw diagrams before beginning to solve the problem.

Attachments:
Attachment A, Pre-Assessment Worksheet
Attachment B, Post-Assessment Worksheet
Attachment C, Classroom Activity Problems
Attachment D, Homework Problems
Solving Problems Involving Chords, Radii, Tangents, Secants and Arcs within the Same Circle – Grade Ten

Attachment A
Pre-Assessment Worksheet

Name _____________________________________

Directions: For questions 1-3, find the measure of each arc or angle. Remember to show your calculations next to each problem.

Given: MN is the diameter of \( \odot Q \), \( \angle PQM = 140^\circ \)

1. \( \angle PNM \)
2. \( PN \)
3. \( \angle PMN \)

For questions 4-5, find the measure of each arc or angle. Remember to show your calculations next to each problem.

Given: FG is tangent to \( \odot A \), \( \angle DCB = 20^\circ \), \( \angle CE = 80^\circ \)

4. \( \angle GCE \)
5. \( \angle BCE \)
Directions: For questions 6-8, find the measure of each segment. Remember to show your calculations next to each problem.

6. If \( w = 10 \), find \( x \).

Calculations:

7. If \( z = 8 \), find \( y \).

8. If \( y = 10 \), find \( z \).

Calculations:
Solving Problems Involving Chords, Radii, Tangents, Secants and Arcs within the Same Circle – Grade Ten

Attachment A (continued)
Pre-Assessment Worksheet

Answer Key

1. 70°  Procedure: \[ \frac{1}{2} (140°) = 70° \]

2. 40°  Procedure: \( (180° - 140°) = 40° \)

3. 20°  Procedure: \[ \frac{1}{2} (40°) = 20° \]

4. 40°  Procedure: \[ \frac{1}{2} (80°) = 40° \]

5. 30°  Procedure: \( 90° - (40° + 20°) = 30° \)

6. 4 \( \) Procedure: \( 2(10) = 5x; x = 4. \)

7. 22 \( \) Procedure: \( (8 + 4) \cdot 4 = 2(y + 2); 48 = 2y + 4; y = 22. \)

8. 2 \( \) Procedure: \( (10 + 2) \cdot 2 = (z + 4) \cdot 4; 24 = 4z + 16; 8 = 4z; z = 2. \)
The average height of the International Space Station is 400 km above the Earth. An astronaut on board the International Space Station sights along a tangent line to Earth, viewing Houston Space Center. If the diameter of the Earth is approximately 12,800 km, how far is the astronaut from the Space Center? Show your calculations; then, explain, in paragraph form, how you found the answer. Note: the drawing is not to scale.

2. An inscribed angle intercepts an arc that is $\frac{1}{5}$ of the circle. Find the measure of the inscribed angle. Remember to show your calculations.
Answer Key

1. \( x = 2300 \text{ km} \quad 400(12800 + 400) = x^2; \quad x = 20\sqrt{13200}; \quad x = \text{approx. 2298 km} \)

The paragraph should include the formula for secant-tangent segment drawn to a circle. The square of the tangent segment is equal to the diameter plus the external segment times the external segment. Solving for the tangent segment, the square root of the diameter plus the external segment times the external segment equals 20 times the square root of 13,200 km, or 2,300 km rounded.

OR

\[
x = \sqrt{(6400 + 400)^2 - 6400^2} \approx 2298
\]

Although not discussed in the lesson, this method is equally valid. Ask students why both methods yield the same answer for \( x \)

2. The angle is 36°

\[
\text{angle} = \left( \frac{1}{5} \right) \left( 360° \right) \left( \frac{1}{2} \right)
\]
Solving Problems Involving Chords, Radii, Tangents, Secants and Arcs within the Same Circle – Grade Ten

Attachment C
Classroom Activity Project

Objective: Students will practice applying theorems relating to inscribed angles, arcs, tangent lines, radii, and chords in the same circle.

1. An arch, in the shape of a circular arc, supports a pipeline (AB) across a river 18 meters wide. Midway, the suspending cable (CD) is 3 meters long. Find the radius of the arch.

2. A puzzle in the form of a quadrilateral is inscribed in a circle. The vertices of the quadrilateral divide the circle into 4 arcs in a ratio of 1:2:5:4. Find the angles of the quadrilateral. After solving this problem, write, in paragraph form, an explanation of how you solved it.

3. A circular trashcan is pushed into a corner. The diameter of the can is 60 cm. Find the distance from the corner (P) to the edge of the can (A).
Solving Problems Involving Chords, Radii, Tangents, Secants and Arcs within the Same Circle – Grade Ten

Attachment C (continued)

Classroom Activity Problems

Answer Key

1. 15 m  
   \[3x = 9^2; \text{ where } x = \text{ chord segment}; \ x = 27; \ \frac{1}{2} (27+3) = \text{ radius}\]

2. 45°, 75°, 135°, 105°  
   \[x + 2x + 5x + 4x = 360^\circ; \ 12x = 360^\circ; \ x = 30^\circ; \]

   OR

   1st angle is \[\frac{1}{2} (30 + 60) = 45^\circ\]

   135°, 75°, 45°, 105°

   2nd angle is \[\frac{1}{2} (60 + 150) = 105^\circ\]

   3rd angle is \[\frac{1}{2} (120 + 150) = 135^\circ\]

   4th angle is \[\frac{1}{2} (120 + 30) = 75^\circ\]

3. \(30\sqrt{2} \ - \ 30 \text{ cm}\)  
   Let \(y = \text{ hypotenuse of right triangle}; \ 30^2 + 30^2 = y^2; \)

   \[30\sqrt{2} = y; 30\sqrt{2} = x + 30; x = 30\sqrt{2} - 30 \text{ cm}\]
1. A conveyor belt is tightly wrapped around circle R, forming a right angle at Q. The circle has a radius of 9 inches. Find the length of the distance from the point on the corner to the closest point on the circle, P. After completing the calculations, write a paragraph explaining how you solved the problem.

2. Two tangent segments to a circle intersect at a point outside the circle. If the sum of their lengths is 35 centimeters, what is the length of each tangent segment?

3. A jeweler wants to engrave an inscribed square within a circle of radius 12 cm. Find the length of the side of the square.

4. An inscribed angle intercepts an arc that is $\frac{1}{8}$ of the circle. Find the measure of the inscribed angle.

5. A soup company wants to package six cans in a box in the shape of an equilateral triangle. The radius of each can is 4 centimeters. Find the length of a side of the package (JH).
Solving Problems Involving Chords, Radii, Tangents, Secants and Arcs within the Same Circle – Grade Ten

Attachment D (continued)

Homework Problems

Answer Key

1. \((9\sqrt{2} - 9)\) inches
   Let \(c =\) hypotenuse; \(9^2 + 9^2 = c^2\); \(\sqrt{162} = c\); \(9\sqrt{2} = c\);
   \(9 + x = c\); \(x = 9\sqrt{2} - 9\) inches

Extension: Find the length of the belt:
\[ l = \frac{3}{4} (2\pi r) + 2r = 60.4 \text{ inches} \]

Paragraph explanation should include the formula for tangent-secant segment and that \(SQ\) is congruent to \(QT\). \(QR\) is equal to radius plus \(PQ\), where \(QR\) is the hypotenuse of right triangle \(RQT\). Using the Pythagorean Theorem and solving for \(QR\), the hypotenuse is found to be \(9\sqrt{2}\). Solving for \(PQ\) by subtracting the radius, the distance is \(9\sqrt{2} - 9\) cm.

2. 17.5
   \(x = 35 - x; 2x = 35; x = 17.5\)

3. \(12\sqrt{2}\) cm
   \(a^2 + a^2 = 24^2; 2a^2 = 576; a^2 = 288, a = 12\sqrt{2}\)

4. 22.5°
   \((\frac{1}{2})(\frac{1}{8})(360°) = (\frac{1}{2})(45°) = 22.5°\)

5. \(\approx 29.9\) cm
   In a \(30°-60°-90°\) triangle, where the side opposite the \(30°\) angle is the radius 4, the leg adjacent is \(4\sqrt{3}\), so
   \(2(4\sqrt{3}) + 4(4) = 29.9\) cm
   \(8\sqrt{3} + 16\)
   \((8\times1.732) + 16\)
   13.856 + 16