

Applying Prime Factorization – Grade Six

Ohio Standards Connection

Number, Number Sense and Operations Standard

Benchmark G

Apply and explain the use of prime factorizations, common factors, and common multiples in problem situations.

Indicator 2

Find and use the prime factorization of composite numbers. For example:

- a. Use the prime factorization to find the greatest common factor (GCF).
- b. Use the prime factorization to find the least common multiple (LCM).
- c. Apply the prime factorization to solve problems and explain solutions.

Mathematical Processes

Benchmark H

Use representation to organize and communicate mathematical thinking and problem solutions.

Lesson Summary:

In this lesson, students use the prime factorization of numbers to find the greatest common factor and least common multiple. Working in small groups, they solve problems using the greatest common factor and least common multiple. Communicating strategies and solutions through oral and written expression help students understand the concept.

Estimated Duration: Two hours

Commentary:

The skill of factoring a number and its application to finding the least common multiple (LCM) or the greatest common factor (GCF) is a key element of future algebraic skills and concepts. Students need to be able to factor with confidence and apply this knowledge in multiple situations. Students must also be able to discern which application, (LCM or GCF,) the situation requires and apply it correctly. This lesson emphasizes both the skill of factoring and its applications in problem situations.

Pre-Assessment:

- Make a list of prime and composite numbers on the board, and use it to review the concept of prime numbers.
- Direct the students working in small groups to find the prime factorization of 36 and 48.
- Ask two students to share their prime factorizations, record them on the board.
- Lead a discussion based on the following questions:
 - a. How do you know when you are finished with a prime factorization?
 - b. Why is it called a prime factorization?
 - c. Can one number have two different prime factorizations? Why or why not?
 - d. Does the order of the factors matter? Why or why not?

Instructional Tip:

Prime factorization, GCF and LCM are prerequisite concepts for this lesson. Help students to realize that (because of the commutative property); order does not matter in regard to the product. However, for the purposes of this lesson, require everyone to record their prime factorization in the same order, smallest factor to greatest factor. Exponents simplify the prime factorization; however, in this lesson direct students to write the prime factorizations without exponents.

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Scoring Guidelines:

Circulate throughout the classroom to observe students as they work. Record the names of students who have little or no previous knowledge of prime factorization. Informally assess the whole class during the discussion part of the pre-assessment. Intervene with students who have difficulty in completing the pre-assessment.

Post-Assessment:

- Direct the students to complete Attachment A, *GCF and LCM Using Prime Factorization Post-Assessment*.
- Have students complete Attachment A individually or with a partner.

Scoring Guidelines:

A sample rubric:

<i>Adequate Understanding</i>	Uses prime factorization, greatest common factor and least common multiple appropriate to the context and provides accurate solutions. Explanation includes clear strategies with mathematical thinking.
<i>Partial Understanding</i>	Provides accurate solutions but omits or provides an incomplete explanation or Provides incorrect solutions with explanations that show some understanding.
<i>Limited Understanding</i>	Provides an inaccurate factorization of some or all of the numbers. Confuses the use of GCF and LCM for the contexts.

Instructional Procedures:

Part One: LCM

1. Pose the following situation to the students:
A discount store sells two types of marbles. Cat’s-eye marbles come in bags of 18. Crystal marbles come in bags of 30. I want to buy the same number of each type of marble.
 - How many of each type of marble must I buy?
 - How many bags of each type must I buy?
 - Predict how many of each type of marble and how many bags you will purchase.
2. Tell the students that they are going to go through the process for answering these questions as a class.
3. Direct students to use a factor tree to find the prime factorization for 18 and 30. Instruct students to reorder their prime factorization in order from least to greatest.
4. Have students check their work with a partner.
5. Record the correct answers on the board. ($18 = 2 \times 2 \times 3$, $30 = 2 \times 3 \times 5$) Model for students how to write the two prime factorizations directly above each other.

$18 = 2 \ 3 \ 3$
$30 = 2 \ 3 \ 5$

6. Explain to the students that they are looking for the “winner” for each factor. Model the following on the board as students complete the work on their papers.

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Instructional Tip:

The winner for each number represented in the prime factorization is the one which has the most representative factors.

- For the factor 2, this is a tie, because each prime factorization has the factor 2 one time. If there is a tie, the number on top wins. Circle the 2 in the prime factorization for 18 and cross out the 2 in the prime factorization of 30.

$$18 = 2 \cdot 3 \cdot 3$$

$$30 = \cancel{2} \cdot 3 \cdot 5$$

- Continue for the factor 3. Eighteen wins, because its prime factorization has two factors of 3. Circle the 3s in the prime factorization of 18, and cross out the 3 in the prime factorization of 30.

$$18 = 2 \cdot 3 \cdot 3$$

$$30 = \cancel{2} \cdot \cancel{3} \cdot 5$$

- Continue for the factor 5. 30 wins because its prime factorization has one 5. Circle the 5 in the prime factorization of 30.

$$18 = 2 \cdot 3 \cdot 3$$

$$30 = \cancel{2} \cdot \cancel{3} \cdot 5$$

- Direct the students to record all of the “winners” in one factor string and to find the product of that factor string.

$$2 \times 3 \times 3 \times 5 = 90$$

Explain to the students that 90 is the LCM for 18 and 30.

- Continue to model using the prime factorization to find the LCM of other numbers. Include examples where the LCM is one of the numbers (such as 120 and 360).
- Direct the students to complete Attachment B, Cicada Census, or Attachment C, Peanut Butter Puzzler. Discuss responses to the attachments.

Part Two: GCF

- Pose the following situation to the students:

You have a bag with 30 crystal marbles and another bag with 42 cat’s-eye marbles. You want to repackage the marbles into smaller bags, and you want each bag to have the same number of each type of marble. You also want as many new bags as you possibly can make.

- How many of each marble will be in the bags?
- How many bags will you be able to make?
- Will you have any marbles left over?

Tell the students that they are going to go through the process for answering these questions as a class.

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11. Direct students to use a factor tree to find the prime factorization for 30 and 42, and reorder their prime factorizations in order from least to greatest.
12. Ask the students to take the prime factorizations and reorder them so the numbers are arranged from least to greatest. Have students check their work with a partner.
13. Record the correct answers on the board. ($30 = 2 \times 3 \times 5$, $42 = 2 \times 3 \times 7$)
14. Direct the students to write the two prime factorizations directly above/below each other on their papers.

$30 = 2 \quad 3 \quad 5$
$42 = 2 \quad 3 \quad 7$

- Explain to students that they are looking for “partners” for each of the factors.
- What numbers from the prime factorization of 30 also appear in the prime factorization of 42? (2 and 3) These are “partners.”
- Circle the two 2’s in a loop. Circle the two 3’s in a loop.
- Do the 5 and the 7 have partners? (No.) Cross out both the 5 and the 7 because they do not have a partner.

$30 = \textcircled{2} \textcircled{3} \cancel{5}$
$42 = \textcircled{2} \textcircled{3} \cancel{7}$

- Direct the students to record each number that had a partner in a factor string and to find the product of that factor string, $2 \times 3 = 6$.

Instructional Tip:

Even though the factors are in “partners,” record each factor only once.

15. Explain to students that six is the GCF for 30 and 42.
16. Continue to model using the prime factorization to find the GCF of other numbers. Review using the prime factorization to find the LCM and GCF.
17. Assign Attachment D, *GCF and LCM*. Once students complete the task, review responses for Attachment D.
18. Facilitate a class discussion focusing on the commonalities and differences between GCF and LCM. Use the following questions to initiate the discussion:
 - What steps (processes) do GCF and LCM have in common?
 - What are the key differences between GCF and LCM?
 - How do you know when to use GCF or LCM? (What are you trying to find?)
 - What are key words that help identify when to use GCF or LCM?

Differentiated Instructional Support:

Instruction is differentiated according to learner needs, to help all learners either meet the intent of the specified indicator(s) or, if the indicator is already met, to advance beyond the specified indicator(s).



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- Allow students to use counters or square tiles to model the problem situations. Provide charts with multiples and factors to use for reference.
- Vary the difficulty of the numbers used in the problems.
- Use the same procedure for finding the GCF or LCM of three or more numbers.
- Place the prime factorizations of two numbers in a Venn diagram. The product of all the factors is the LCM; the product of the factors in just the intersection is the GCF. Ask students to explain why this works.

Extensions:

- Research cicada broods to determine when periodical cicadas will come to a given area.
- Use contexts around transportation schedules. For example:
The red line bus takes 20 minutes to complete its route from the time it leaves from and returns to the station. The blue line bus takes 25 minutes to complete its route from the time it leaves from and returns to the station. If both buses begin their routes at 6:00 a.m., how many times throughout the day will they meet at the station at the same time, if the busses stop running at 11:00 p.m.? When is the first time they will meet? Make a chart to show the times both buses are at the station at the same time.

Home Connection:

Teach someone at home how to use the prime factorization of two numbers to find the GCF and the LCM.

Materials and Resources

The inclusion of a specific resource in any lesson formulated by the Ohio Department of Education should not be interpreted as an endorsement of that particular resource, or any of its contents, by the Ohio Department of Education. The Ohio Department of Education does not endorse any particular resource. The Web addresses listed are for a given site's main page, therefore, it may be necessary to search within that site to find the specific information required for a given lesson. Please note that information published on the Internet changes over time, therefore the links provided may no longer contain the specific information related to a given lesson. Teachers are advised to preview all sites before using them with students.

For the student: markers or crayons for each group, sheet of chart paper for each group, counters or square tiles, and calculators (optional)

Vocabulary:

- factor
- greatest common factor (GCF)
- least common multiple (LCM)
- multiple
- prime factorization

Technology Connection:

Use a calculator to check the prime factorizations for each number.



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Research Connections:

Cawletti, Gordon. *Handbook of Research on Improving Student Achievement*. Arlington, Va.: Educational Research Service, 1999.

Marzano, Robert J., Jane E. Pollock and Debra Pickering. *Classroom Instruction that Works: Research-Based Strategies for Increasing Student Achievement*, Alexandria, Va.: Association for Supervision and Curriculum Development, 2001.

Pask, Gordon. *Conversation, Cognition and Learning*. New York: Elsevier, 1975.

Attachments:

Attachment A, *GCF and LCM Using Prime Factorization, Post-Assessment*

Attachment B, *Cicada Census*

Attachment C, *Peanut Butter Puzzler*

Attachment D, *GCF and LCM*



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Attachment A GCF and LCM Using Prime Factorization, Post-Assessment

Name _____ Date _____

Directions: Read the scenario and complete the tasks.

Your best friend has a cold and missed math class. Write a set of directions for your friend to show how to find the GCF of 120 and 136 and the LCM of 16 and 18, using the prime factorization of each number. Your directions should include a complete solution so your friend can see how you used the prime factorization. Include two or three tips to help your friend avoid confusion.



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Attachment B Cicada Census

Name _____ Date _____

Sue dislikes cicadas! She is happy when she finds out that the bugs only come out every 17 years in Ohio. Her mom tells Sue that when she was a girl growing up in Illinois, they had some cicadas that came out every 17 years and others that came out every 13 years. Sue's mom remembers that in the summer of 1963, both the 17-year cicadas and the 13-year cicadas came out. When is the next time that both of these groups will come out together? Use the prime factorization of 13 and 17 to help you solve this problem.



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Attachment C Peanut Butter Puzzler

Name _____ Date _____

Don loves peanut butter and jelly sandwiches. One day while he was eating, he notice that each jumbo jar of peanut butter has 72 servings, but the jelly jar has only 40 servings. If he opened the jars on the same day and used exactly one serving each day, how many days would it take until he emptied a peanut butter jar and a jelly jar on the same day? Use prime factorization.

